Power Systems Stability through Piecewise Monotonic Data Approximations – Part 2: Adaptive Number of Monotonic Sections and Performance of L1PMA, L2WPMA, and L2CXCV in Overhead Medium-Voltage Broadband over Power Lines Networks

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This second paper investigates the role of the number of monotonic sections during the mitigation of measurement differences in overhead medium-voltage broadband over power lines (OV MV BPL) transfer functions. The performance of two well-known piecewise monotonic data approximations that are based on the number of monotonic sections (*i.e.*, L1PMA and L2WPMA) is assessed in comparison with the occurred measurement differences and L2CXCV, which is a piecewise monotonic data approximation without considering monotonic sections.
The contribution of this paper is double. First, further examination regarding the definition of the optimal number of monotonic section is made so that the accuracy of L1PMA can be significantly enhanced. In fact, the goal is to render piecewise monotonic data approximations that are based on the optimal number of monotonic sections as the leading approximation against the other ones without monotonic sections. Second, a generic framework concerning the definition of an adaptive number of monotonic sections is proposed for given OV MV BPL topology.

Keywords: *Smart Grid; Intelligent Energy Systems; Broadband over Power Lines (BPL) networks; Power Line Communications (PLC); Faults; Power System Stability; Fault Analysis; Fault Identification and Prediction; Distribution Power Grids*

1. Introduction

More than 100 million BPL devices with annual growth rate of 30% have already been deployed, being able to deliver high-bandwidth applications (*e.g.*, HD video streaming and VoIP) with data rates that exceed 1Gbps [1]-[3]. However, higher data rates can be achieved if the inherent BPL deficiencies, such as high and frequency-selective channel attenuation, noise, faults and measurement differences, are counterbalanced [4]-[9].

As the determination of channel attenuation and the identification of faults and measurement differences are concerned [10], the well-established hybrid method is employed as the suitable theoretical basis for describing BPL signal propagation and

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transmission [4]-[8], [11]-[23]. Actually, the hybrid method, which is employed to examine the behavior of various multiconductor transmission line (MTL) structures, gives as output the corresponding transfer function for given OV MV BPL network topology, OV MV MTL configuration, and applied coupling scheme.

Although the hybrid method is notably accurate, a number of practical reasons and “real-life” conditions create measurement differences between experimental and theoretical results influencing the broadband performance of BPL networks and affecting the overall network design. On the basis of six measurement difference categories, which are analyzed in [21]-[23], piecewise monotonic data approximations can be applied in order to mitigate the measurement differences and restore the broadband performance [10], [22]-[31]. So far, three piecewise monotonic data approximations have been examined that are divided into two groups: (i) *Piecewise monotonic data approximations with predefined monotonic sections*: L1PMA and L2WPMA have been defined in [26], [27], [32] and their performance regarding the mitigation of measurement differences in transmission and distribution BPL networks has been assessed in [10], [21]-[23]. Already been identified, the performance of L1PMA and L2WPMA mainly depends on the predefined number of monotonic sections. In fact, the best performance against measurement differences is achieved when a specific number of monotonic sections is identified and applied; and (ii) *Piecewise monotonic data approximations without predefined monotonic sections*: L2CXCV has been defined in [33] and its performance concerning the mitigation of measurement differences in transmission BPL networks has been assessed in [10]. L2CXCV performance depends neither on user- nor on computer-defined number of monotonic sections. In accordance with [10], the right selection of the number of monotonic sections plays the key role during the application of L1PMA and L2WPMA and the comparative benchmark analysis between the aforementioned groups. In this companion paper, the selection of an adaptive number of monotonic sections is extended from the traditional definition of the optimal number of monotonic sections [10], [21]-[23] to the proposed adaptive one with regards to the maximization of the percent error sum (PES), which is treated as a metric of the mitigation performance of piecewise monotonic data approximations against measurement differences [10], [21].

The rest of this paper is organized as follows: In Sec. II, a brief presentation of L1PMA, L2WPMA, and L2CXCV is given as well as the suitable metrics of PES and fault PES, which are applied in order to identify the optimal number of monotonic sections. Sec. III discusses the simulations of various OV MV BPL networks intending to identify the generic framework concerning the definition of an adaptive number of monotonic sections to mitigate the occurred measurement differences. Sec. IV concludes this paper.

2. Brief Presentation of L1PMA, L2WPMA, L2CXCV and PESs

A set of piecewise monotonic data approximations has already been comparatively benchmarked concerning its mitigation behavior against measurement differences during the OV MV BPL transfer function determination [10], [21]-[23]. L1PMA, L2WPMA, and L2CXCV have been assessed with regards to: (i) their relative PES; and (ii) their PES against fault PES. As already been shown in [10], piecewise monotonic data approximations that are based on the optimal number of monotonic sections (*i.e.*, L1PMA and L2WPMA) better cope with the measurement differences in

OV MV BPL topologies of intense multipath environments (*i.e.*, urban topologies) whereas piecewise monotonic data approximations without predefined monotonic sections (*i.e.*, L2CXCV) better deal with the measurement differences of OV MV BPL topologies of "quiet" multipath environments (*i.e.*, suburban, rural, and "LOS" topologies). However, significant PES improvement of L1PMA and L2WPMA can be achieved if a careful study concerning the optimal number of monotonic sections is carried out.

2.1 Piecewise Monotonic Data Approximations with Predefined Monotonic Sections (L1PMA and L2WPMA)

L1PMA and L2WPMA exploit their piecewise monotonicity property by decomposing BPL coupling transfer function data into separate monotonous data sections between adjacent turning points (primary extrema). Then, L1PMA and L2WPMA separately handle the monotonous sections by proposing suitable regression approximation [21]-[23]. In general terms, L1PMA and L2WPMA software receives as inputs the measured OV MV BPL coupling transfer function data, the measurement frequencies and the number of monotonic sections (*i.e.*, either user- or computer-defined) and gives as outputs the optimal primary extrema and the best fit of the measured OV MV BPL coupling transfer function data. The mitigation performance of L1PMA and L2WPMA against measurement differences mainly depends on the number of monotonic sections while the best performance is achieved when a critical number of monotonic sections is adopted.

2.2 Piecewise Monotonic Data Approximations without Predefined Monotonic Sections (L2CXCV)

L2CXCV smooths the OV MV transfer function data in the least square error sense by assuming one sign change in the second divided differences of the smoothed values [33]. In contrast with L1PMA and L2WPMA, the number of monotonic sections is neither user- nor computer-defined since L2CXCV computes the required fit by solving a strictly convex quadratic programming problem for each set. Since L2CXCV better deals with the measurement differences of OV MV BPL topologies of "quiet" multipath environments, it acts as the benchmark for the evaluation of the improved L1PMA (see Sec. III).

2.3 PES, Fault PES and ΔPES

As it has already been mentioned in [10] and [21], to evaluate the mitigation performance of the piecewise monotonic data approximation methods against the presented measurement differences, the performance metrics of PES, fault PES, and ΔPES are applied.

More specifically, PES expresses as a percentage the total sum of the relative differences between the approximated coupling transfer function and the theoretical coupling transfer function for all the used frequencies, namely

$$PES = 100\% \cdot \frac{\sum_{i=1}^{u} \left| \overline{\overline{\mathbf{H}^{\mathrm{WtG}}}}(f_i) - \mathbf{H}^{\mathrm{WtG}}(f_i) \right|}{\sum_{i=1}^{u} \left| \mathbf{H}^{\mathrm{WtG}}(f_i) \right|} \tag{1}$$

where $\mathbf{H}^{\mathrm{WtG}}(f_i)$ is the $u\times1$ theoretical OV MV BPL coupling transfer function column vector for given WtG coupling scheme and measurement frequency f_i, i=1,…,u, $\overline{\mathbf{H}^{\mathrm{WtG}}}(f_i)$ is the respective measured OV MV BPL coupling transfer function, $\overline{\overline{\mathbf{H}^{\mathrm{WtG}}}}(f_i)$ is the respective approximated OV MV BPL coupling transfer function and u is the number of the assumed flat-fading subchannels in the examined frequency band of operation. With respect to eq. (1), to evaluate the mitigation efficiency of the piecewise monotonic data approximation methods towards the measurement differences, PES of eq.(1) is compared against the fault PES that is given by

$$PES_{\mathrm{fault}} = 100\% \cdot \frac{\sum_{i=1}^{u}\left|\overline{\mathbf{H}^{\mathrm{WtG}}}(f_i) - \mathbf{H}^{\mathrm{WtG}}(f_i)\right|}{\sum_{i=1}^{u}\left|\mathbf{H}^{\mathrm{WtG}}(f_i)\right|} \tag{2}$$

Indeed, with reference to eq. (1) and (2), the proposed ΔPES metric that is determined by

$$\Delta PES = -\left(PES - PES_{\mathrm{fault}}\right) \tag{3}$$

expresses the difference between PES and fault PES. ΔPES achieves to assess the mitigation efficiency of the examined piecewise monotonic data approximation method; if ΔPES is positive then the piecewise monotonic data approximation method counterbalances the measurement differences. Note that the measurement differences, which are applied during the simulations of Sec. III, follow continuous uniform distributions (CUDs) with variable maximum value a_{CUD} as already done in [10], [21].

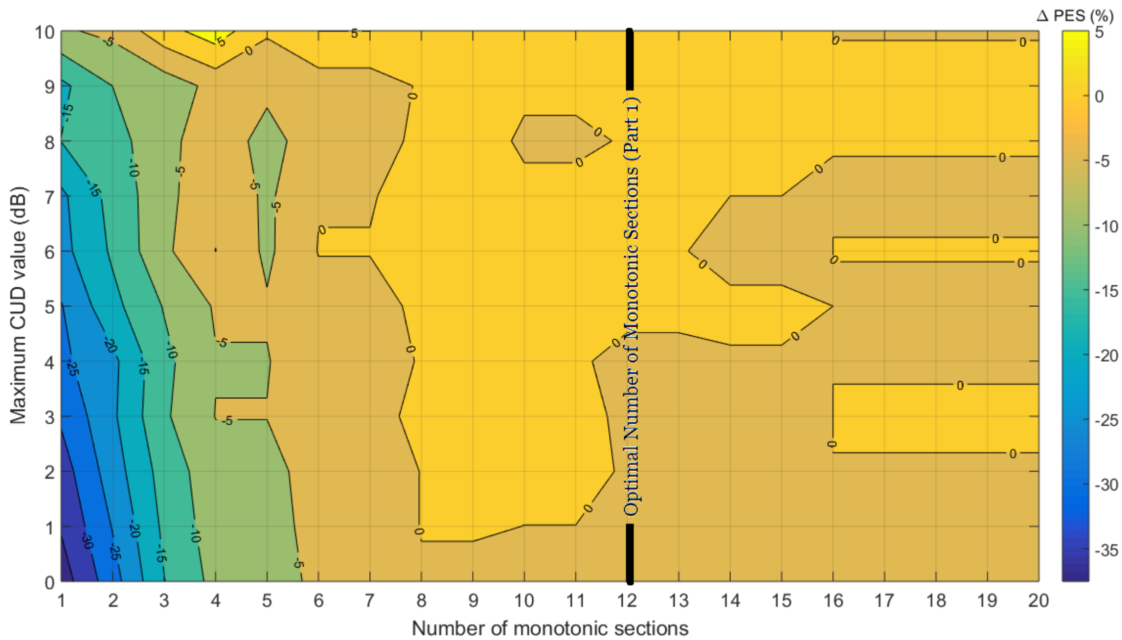
3. Numerical Results and Discussion

Various configurations of OV MV BPL networks are simulated with the purpose of assessing the mitigating performance of the piecewise monotonic data approximation methods of this paper against the occurred measurement differences. In fact, different OV MV BPL topologies and WtG coupling schemes are tested for various maximum CUD values.

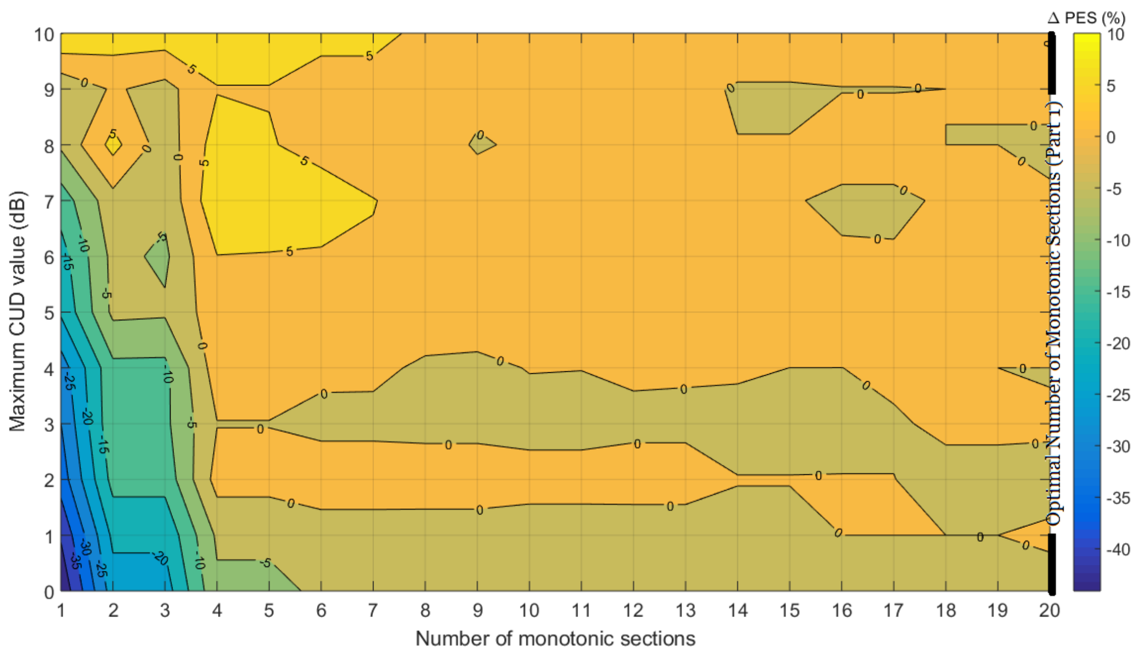
As the simulation specifications are regarded, those are the same with [10]; the BPL frequency range and flat-fading subchannel frequency spacing are assumed to be equal to 1-30MHz and 1MHz, respectively. Therefore, the number of subchannels in the examined frequency range is equal to 30. The OV MV BPL topologies, which have been presented in Sec. IIB of [10], are also used in this paper while all the available WtG coupling schemes, say WtG^i, i=1,…,3, that may be supported by the OV MV MTL configurations are investigated during the following simulations. Finally, the maximum CUD values that are examined range from 0dB to 10dB with 1dB step.

3.1 Optimal Number of Monotonic Sections and Different OV MV BPL Topologies and Coupling Schemes

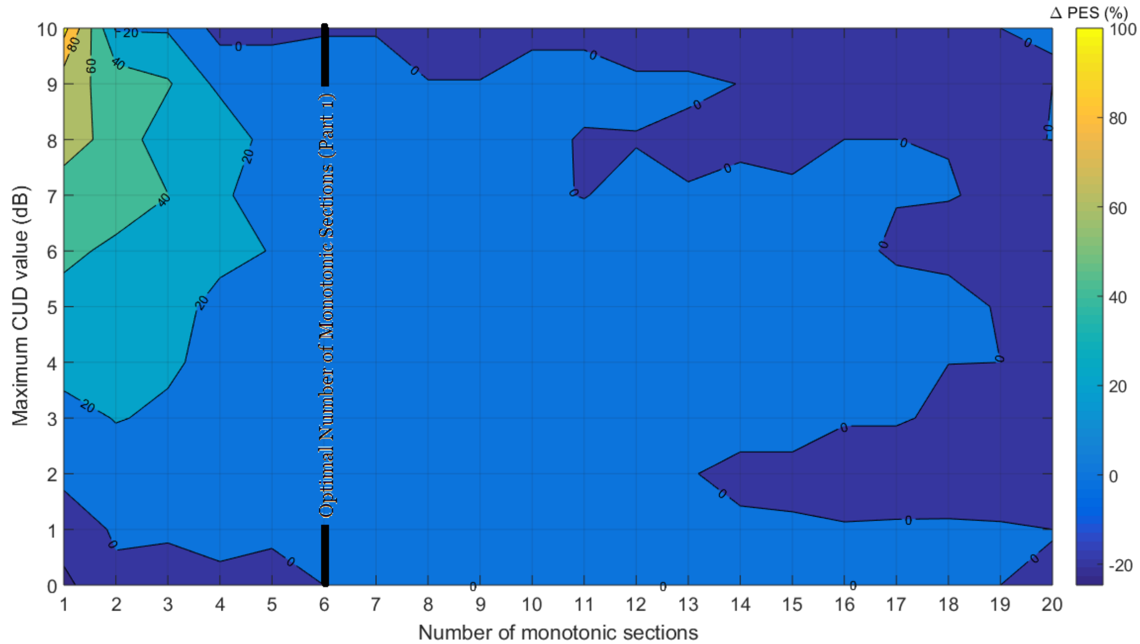
In Fig. 1(a), ΔPES is plotted versus the maximum CUD value and the number of monotonic sections. In this figure, urban case is examined when WtG^1 coupling scheme and L1PMA are applied. The optimal number of monotonic sections that is analytically reported in [10], which is equal to 12 for the indicative urban case, is also drawn in the figure as a vertical line. In Figs. 1(b)-(d), same plots with Fig. 1(a) are given but for the case of suburban, rural, and "LOS" case. Figs. 1(e)-(h) are the same with the respective Figs. 1(a)-(d) but for the application of L2WPMA. In Figs. 2(a)-(h) and Figs. 3(a)-(h), same plots with Figs. 1(a)-(h) are drawn but for WtG^2 and WtG^3 coupling schemes, respectively.



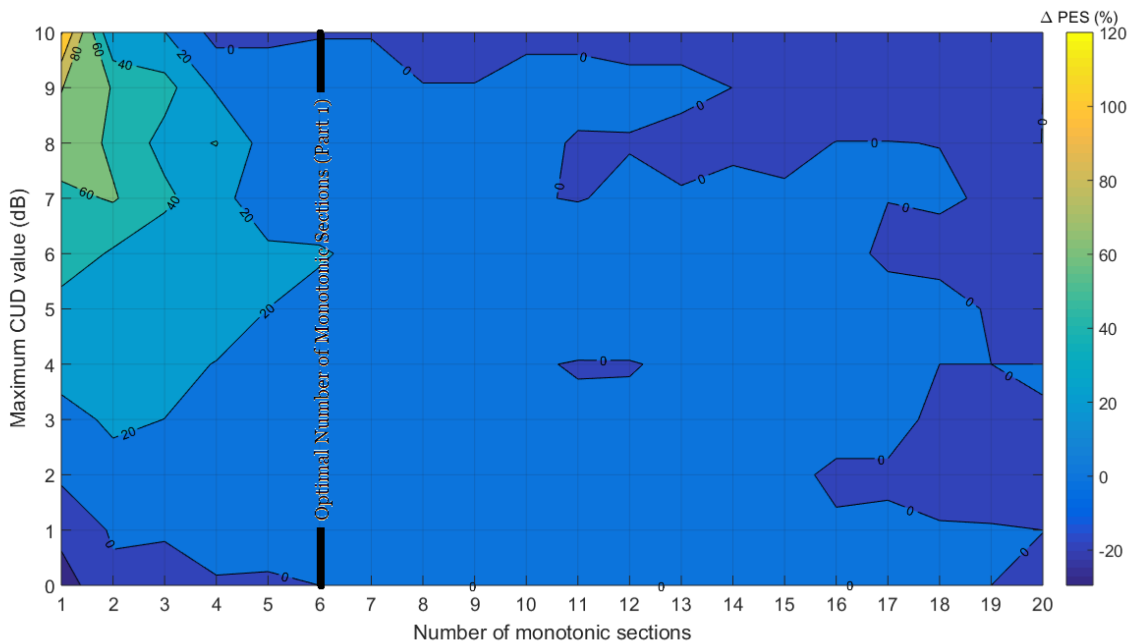
(a)



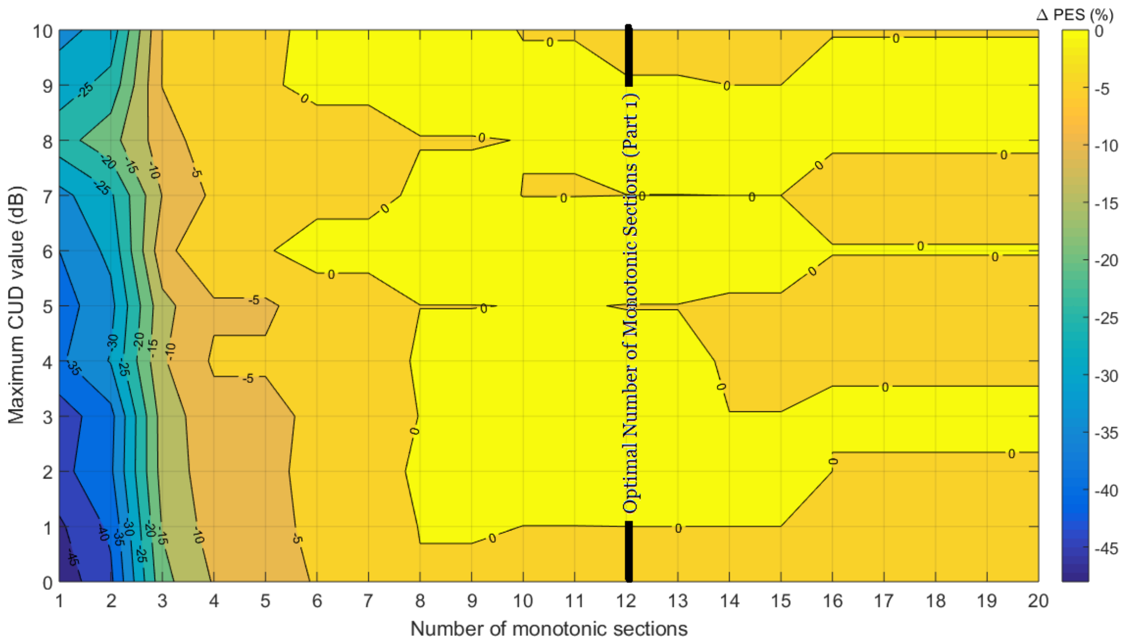
(b)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

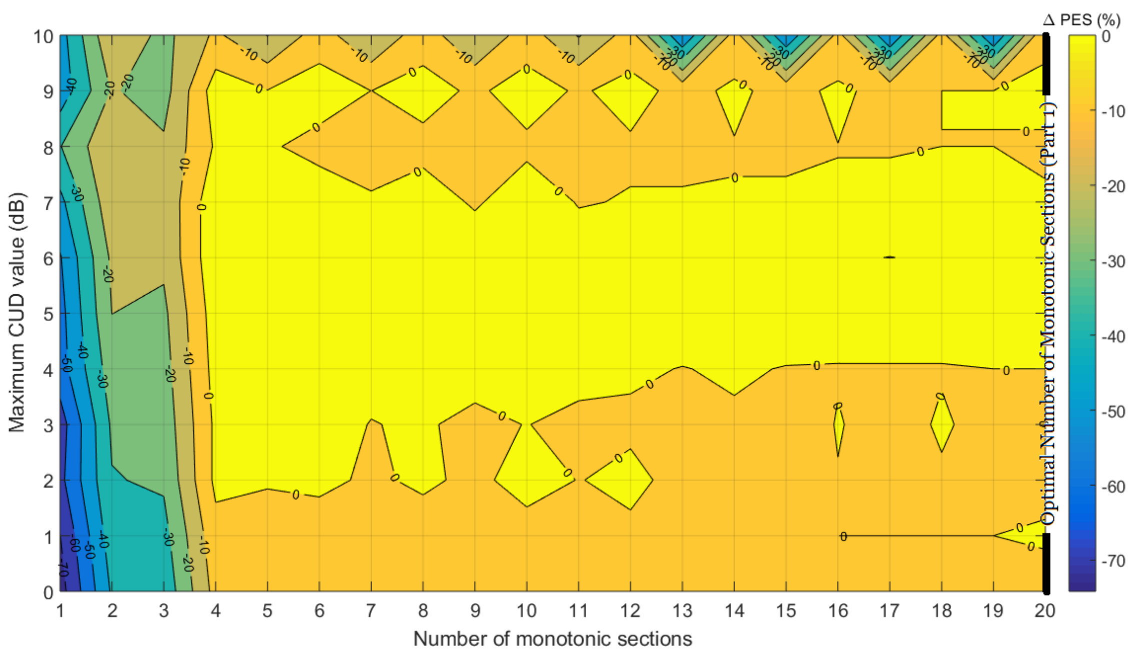
(c)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

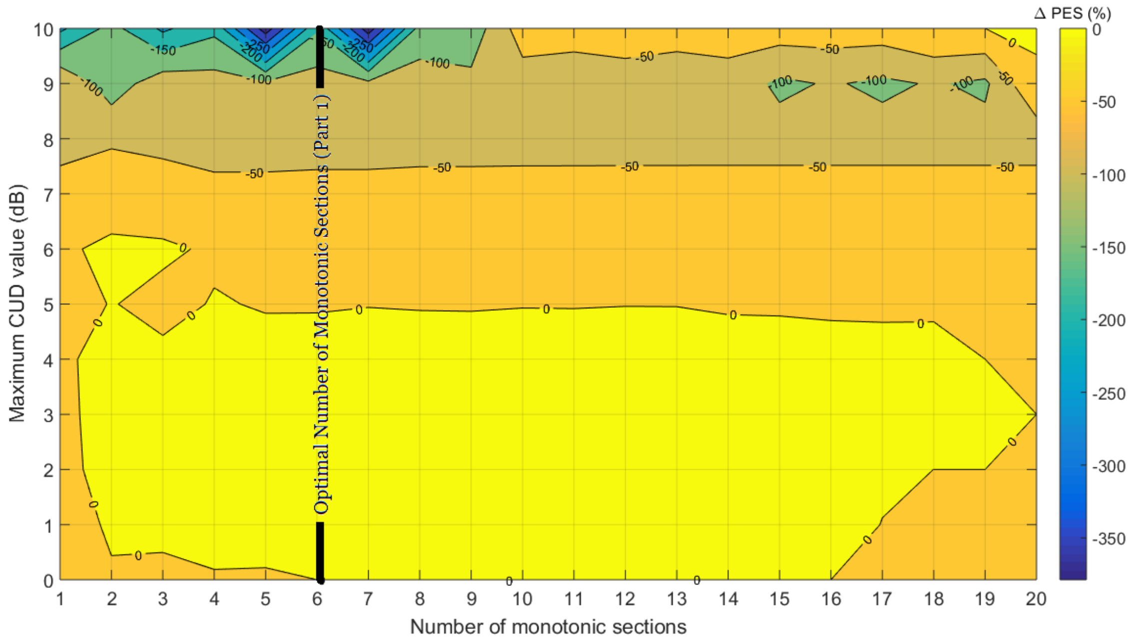
(d)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(e)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(f)



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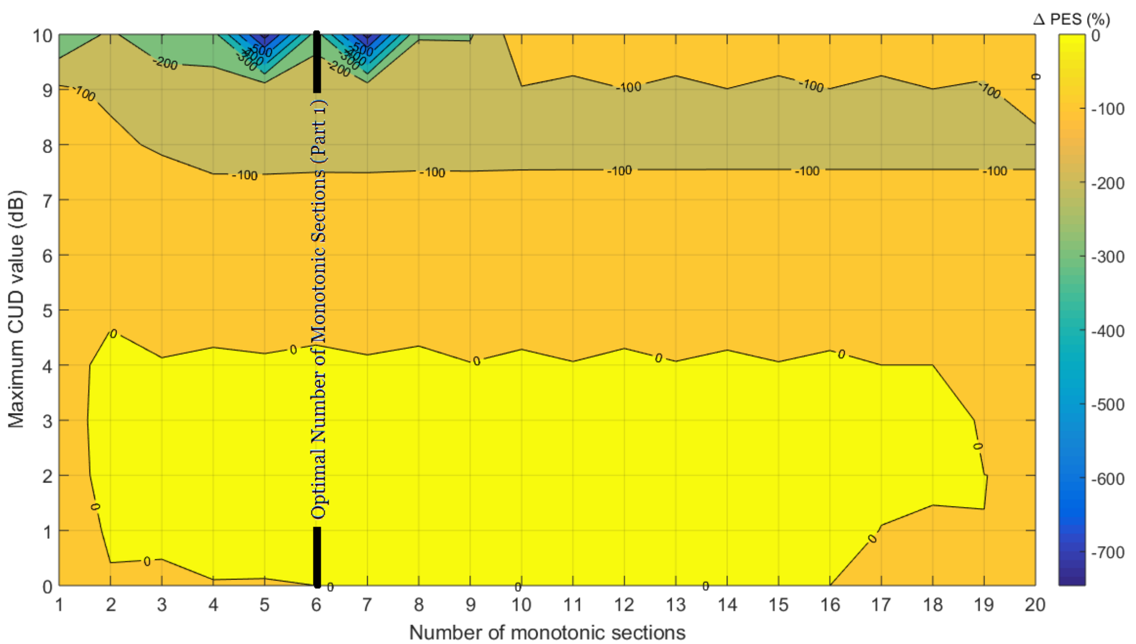
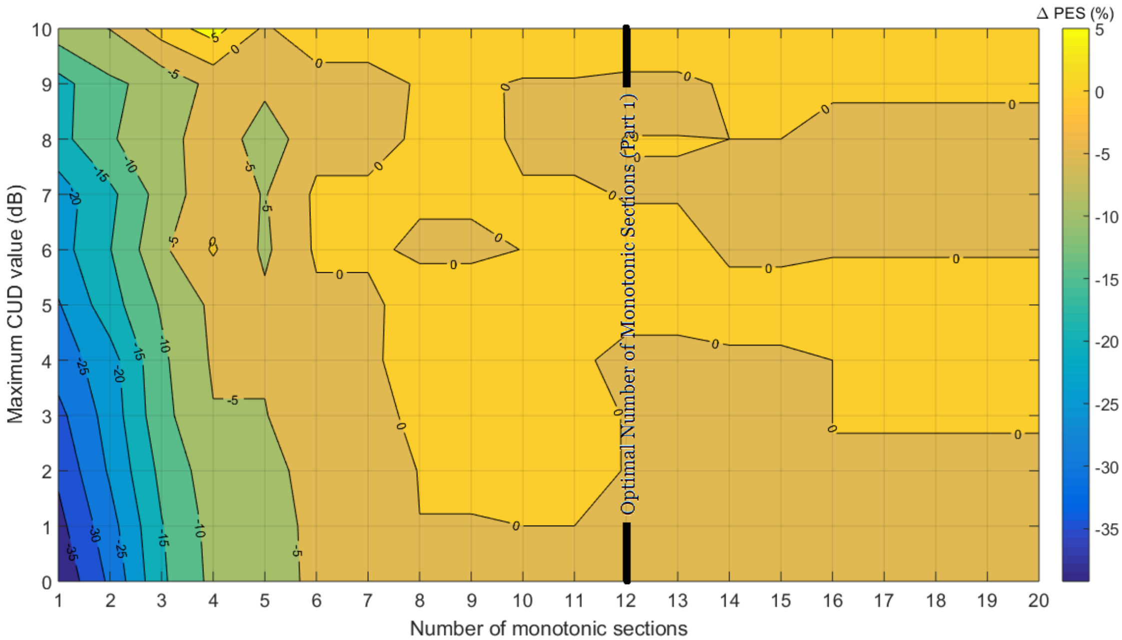
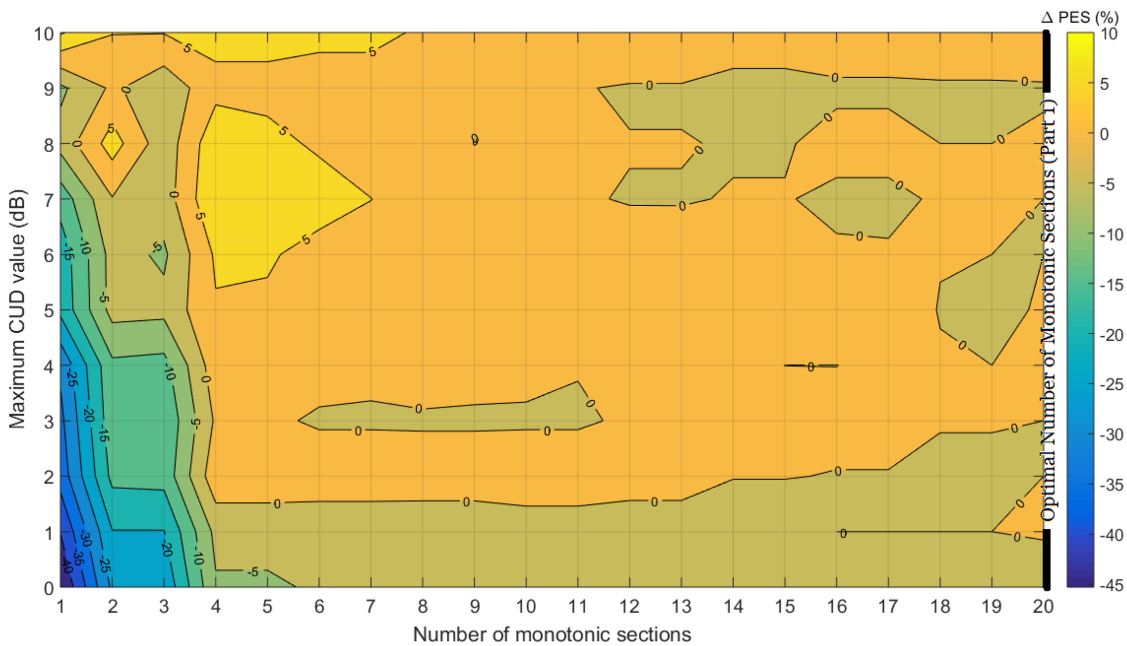


(h)

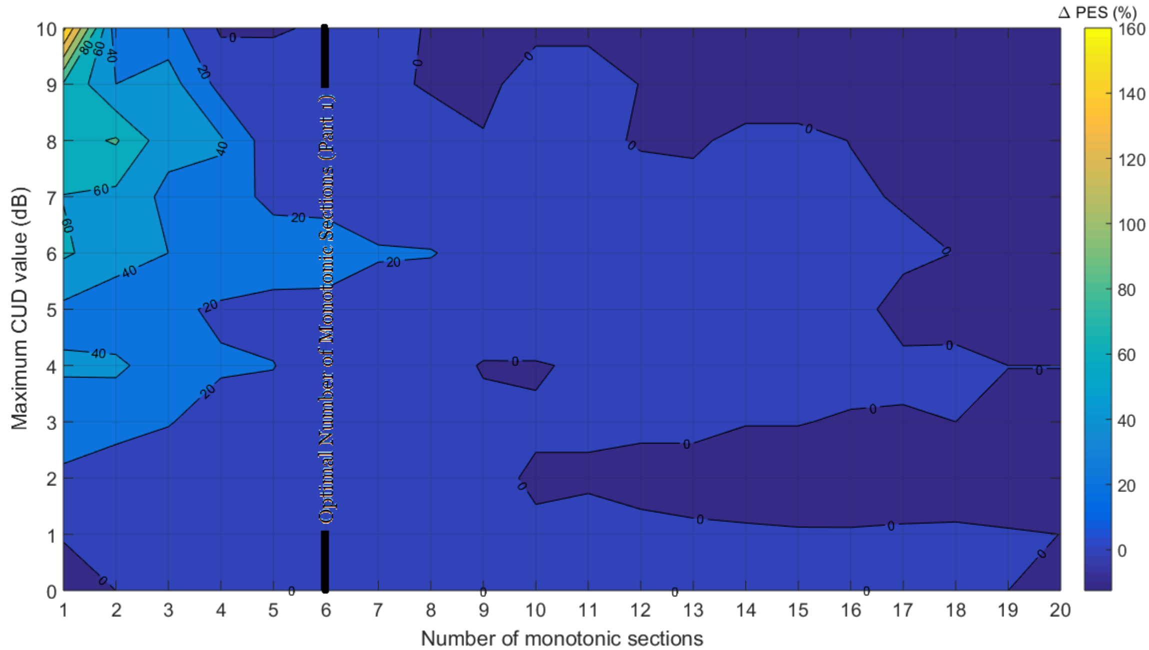
Figure 1. ΔPES is plotted for various indicative OV MV BPL topologies with respect to the maximum CUD value and number of monotonic sections (WtG[1] coupling scheme is applied and the vertical line of the optimal number of monotonic sections is shown). (a) Urban case / L1PMA. (b) Suburban case / L1PMA. (c) Rural case / L1PMA. (d) “LOS” case / L1PMA. (e) Urban case / L2WPMA. (f) Suburban case / L2WPMA. (g) Rural case / L2WPMA. (h) “LOS” case / L2WPMA.


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

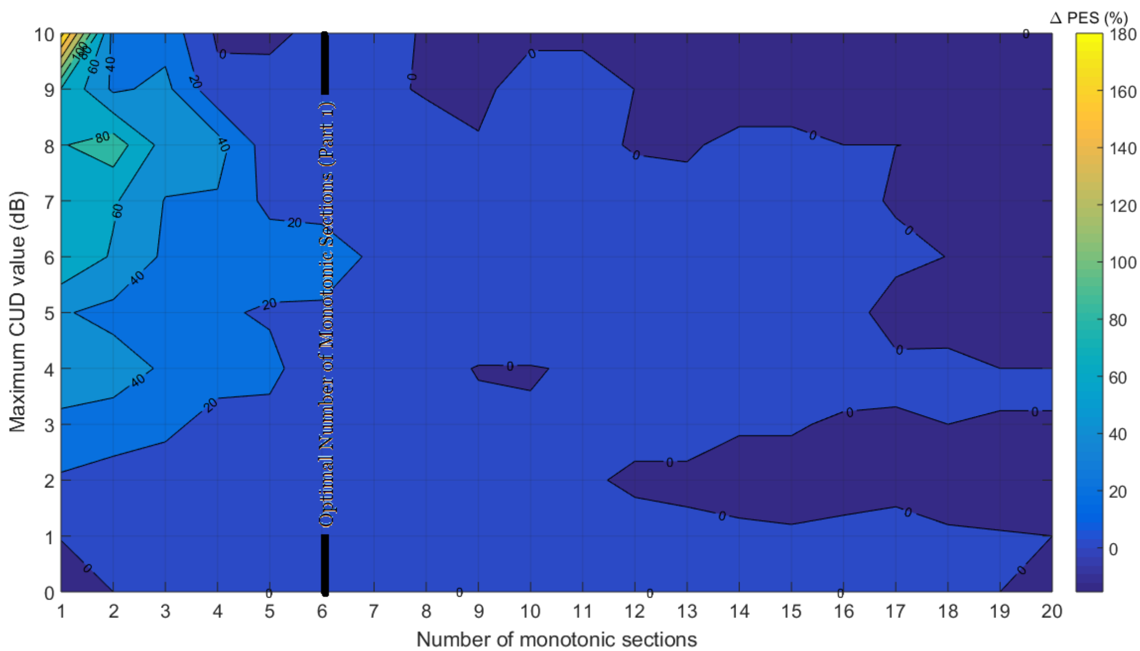
(a)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

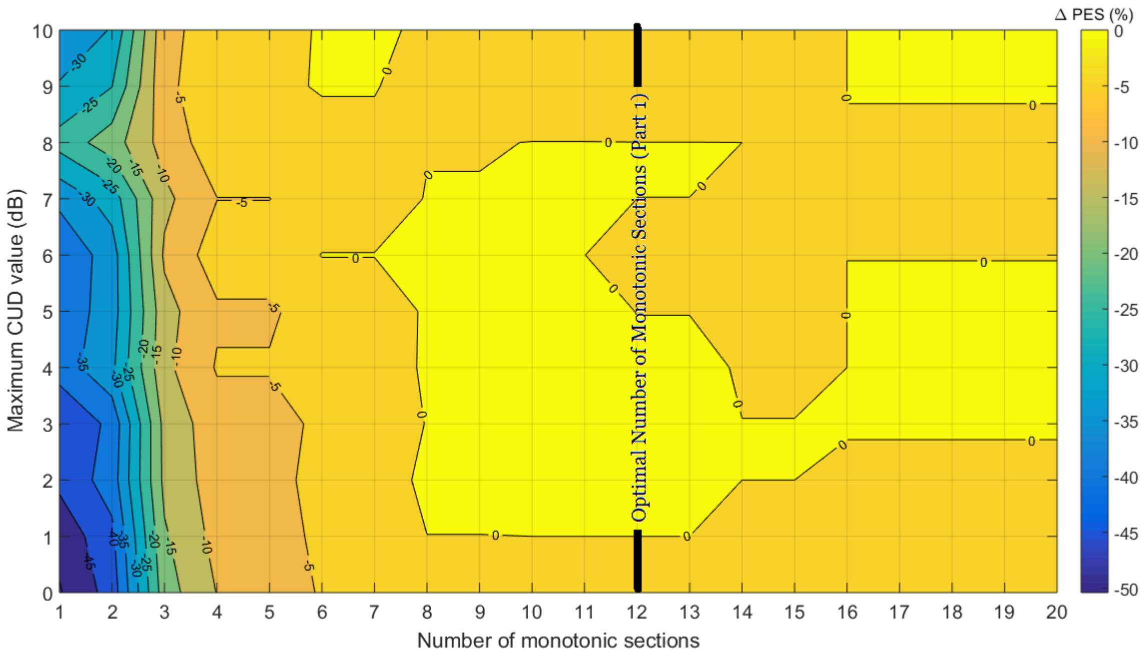
(b)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

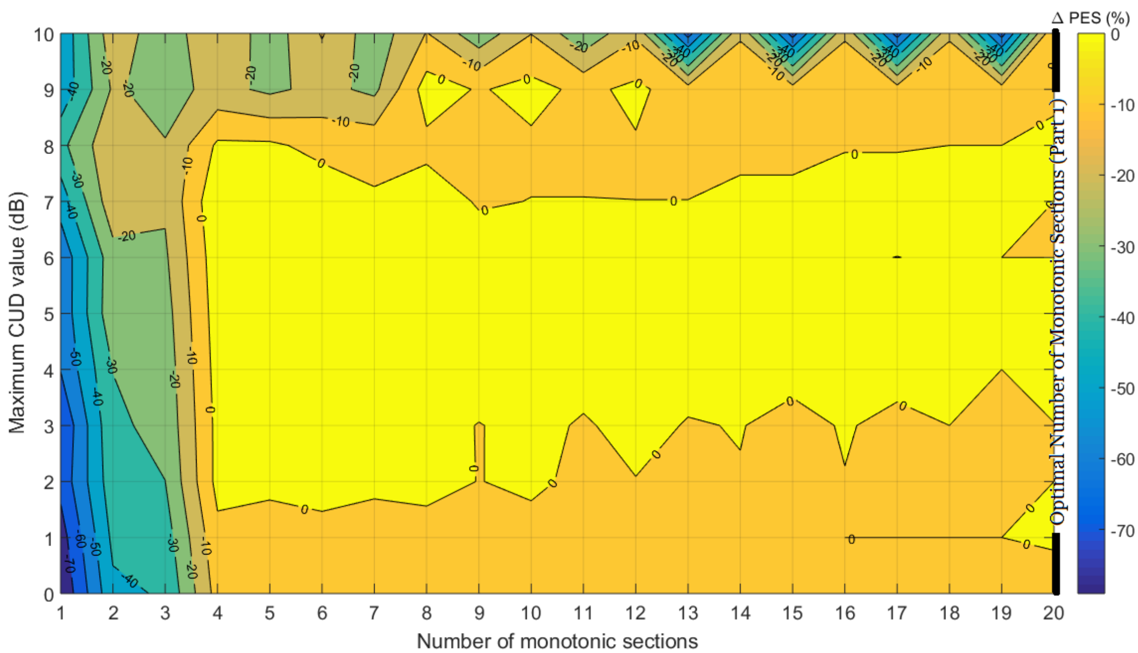
(c)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(d)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(e)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(f)

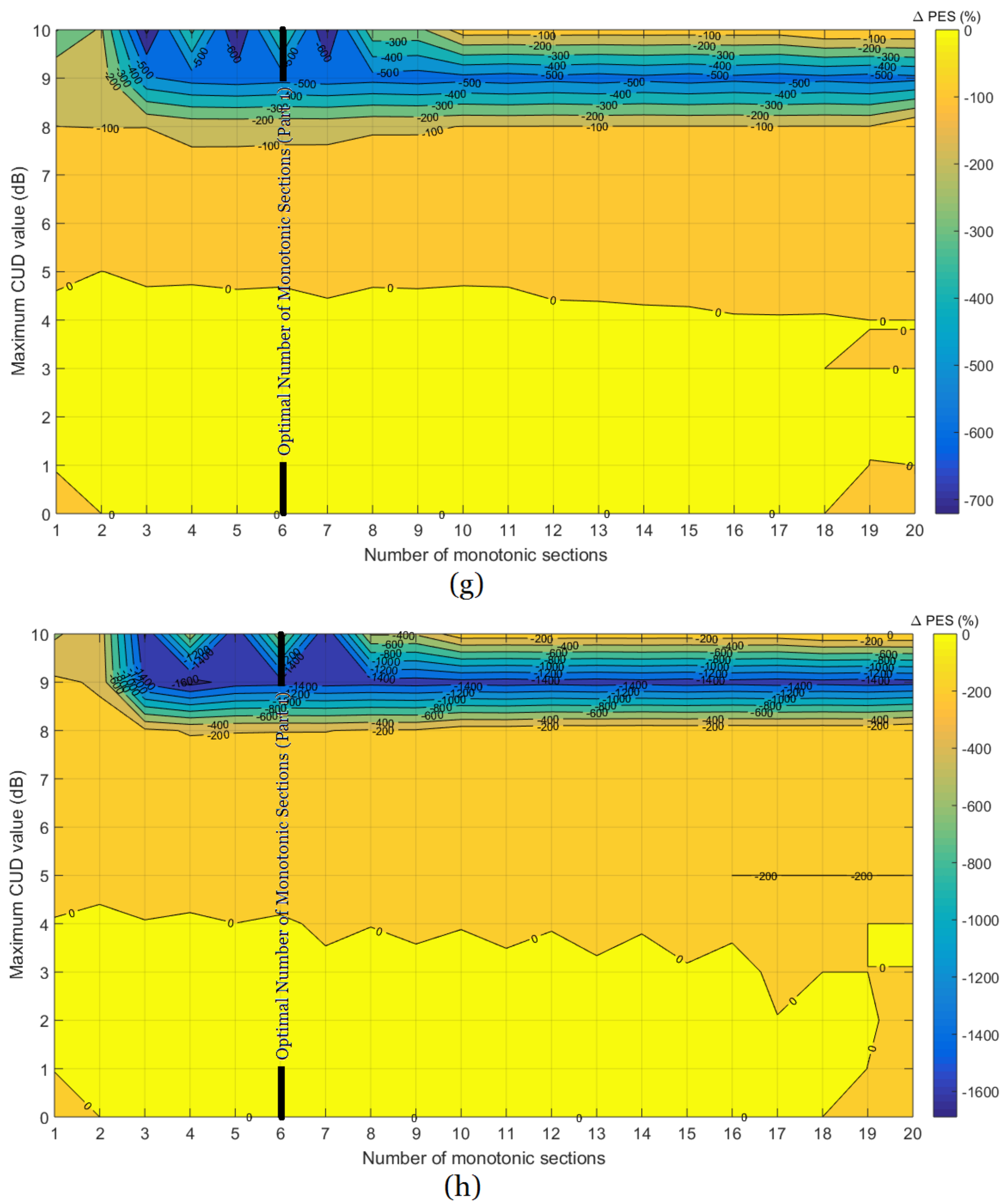
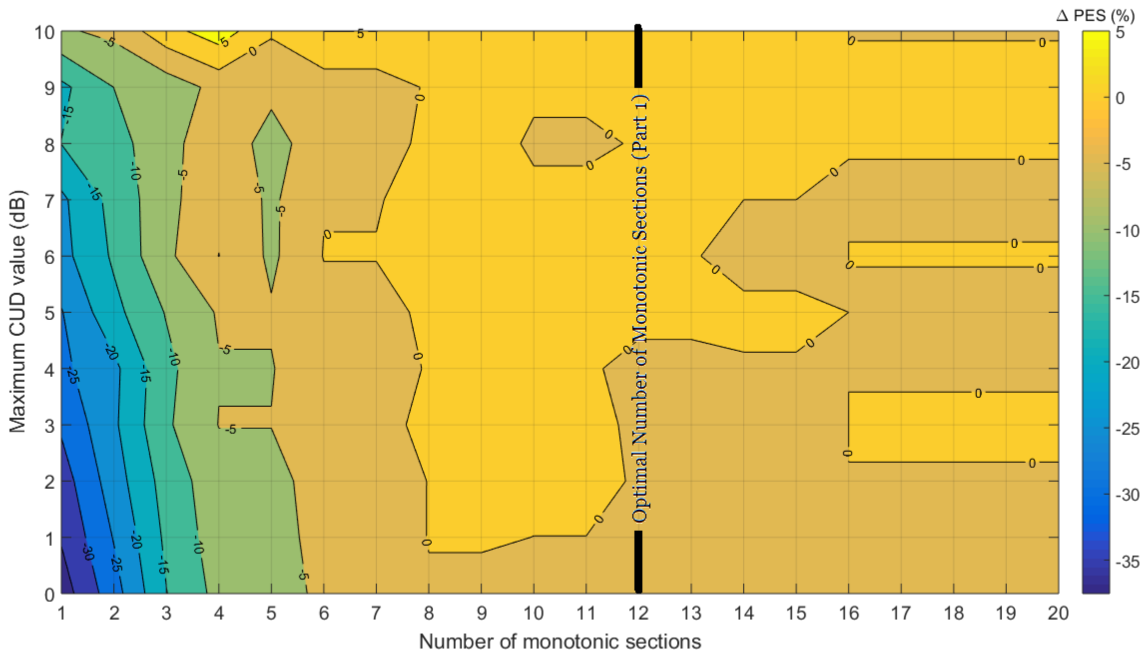
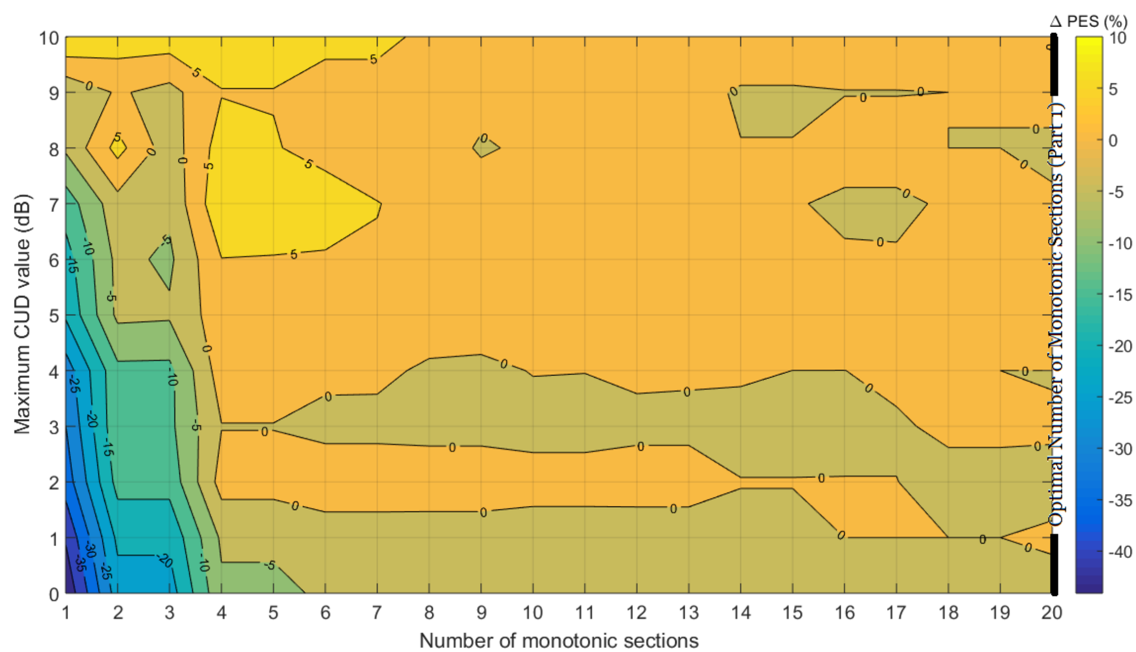


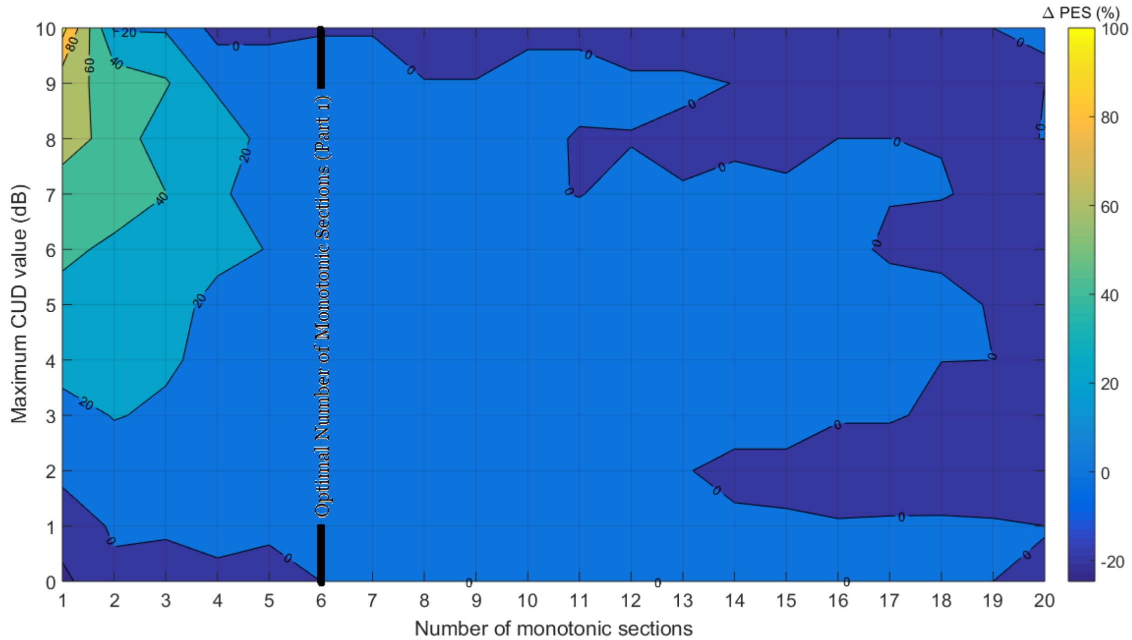
Figure 2. Same with Fig. 1 but for the WtG² coupling scheme.

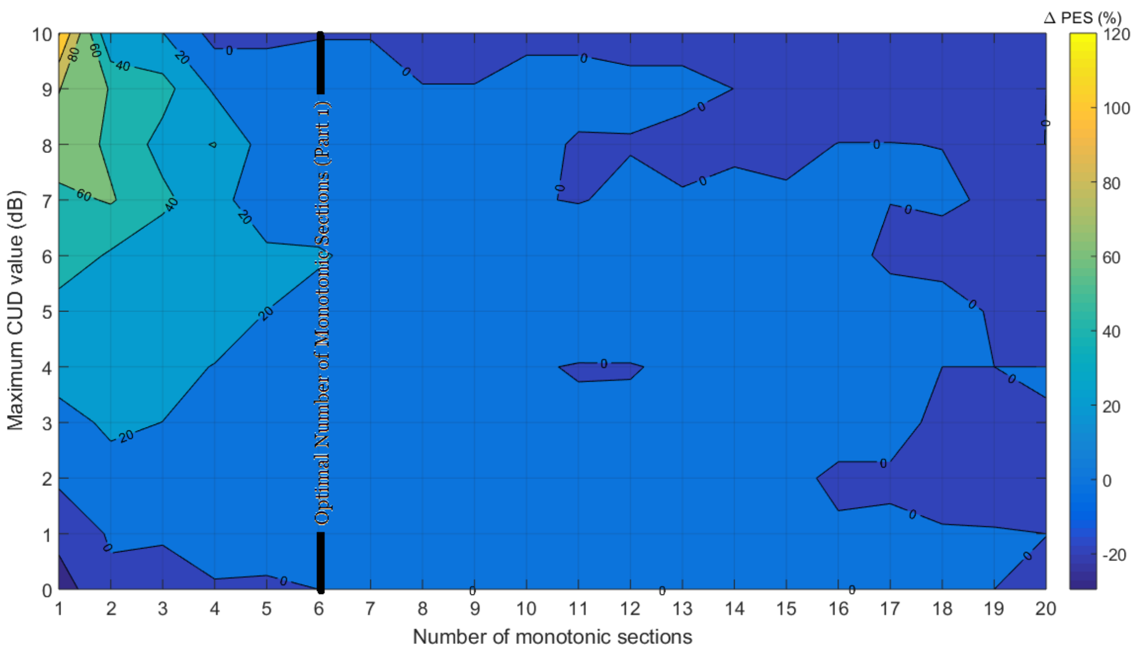

Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

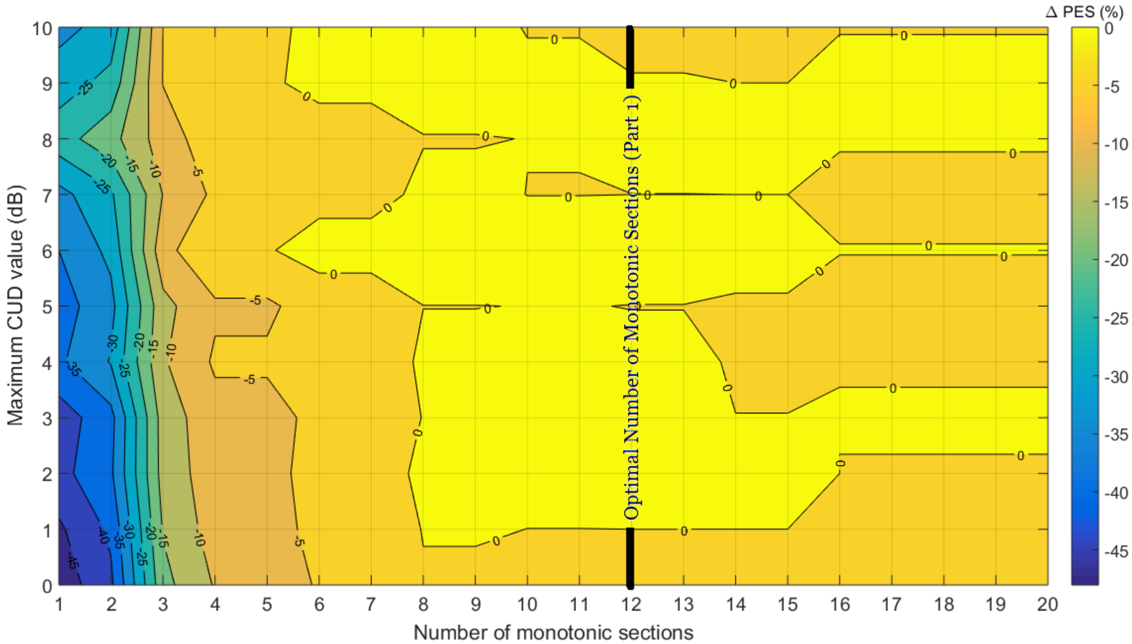
(a)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(b)

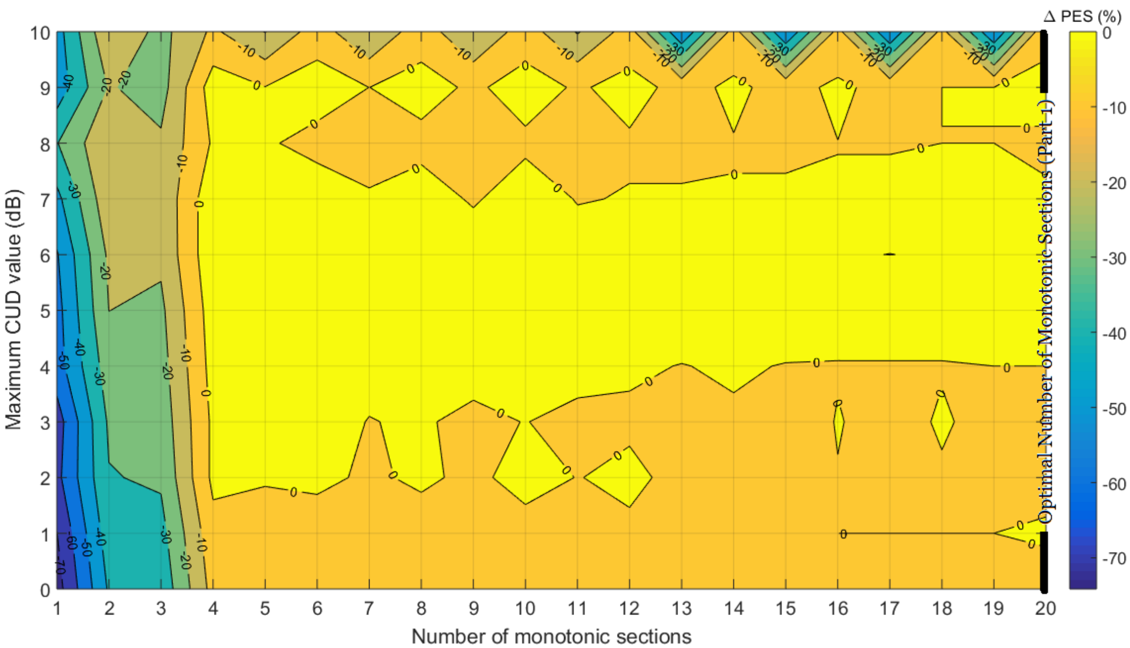

Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)
(c)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)
(d)



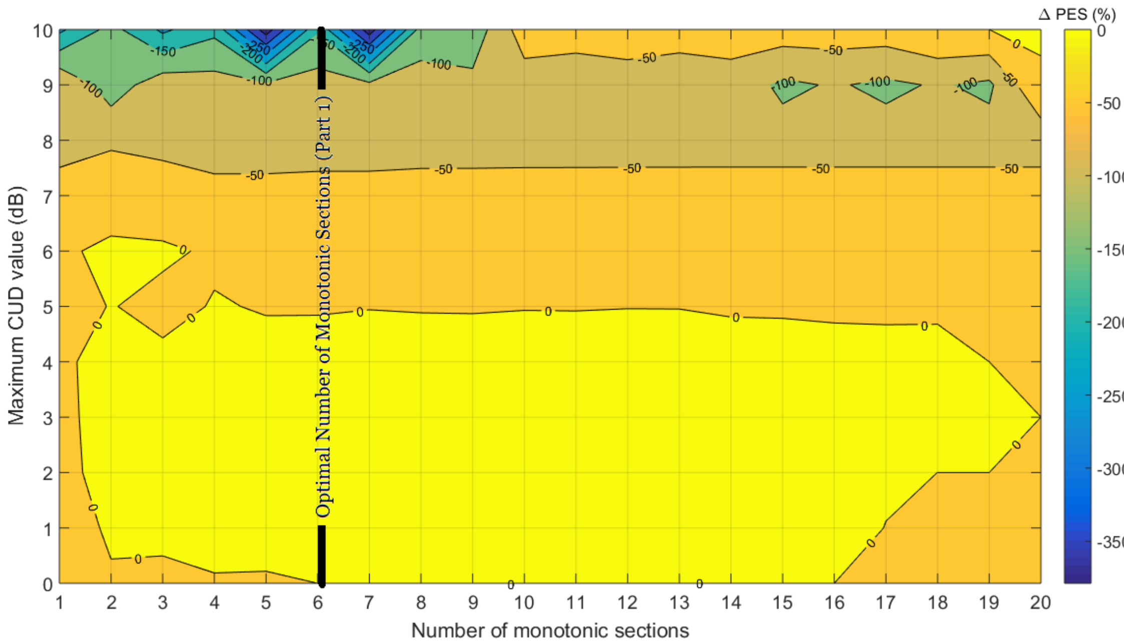
Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(e)

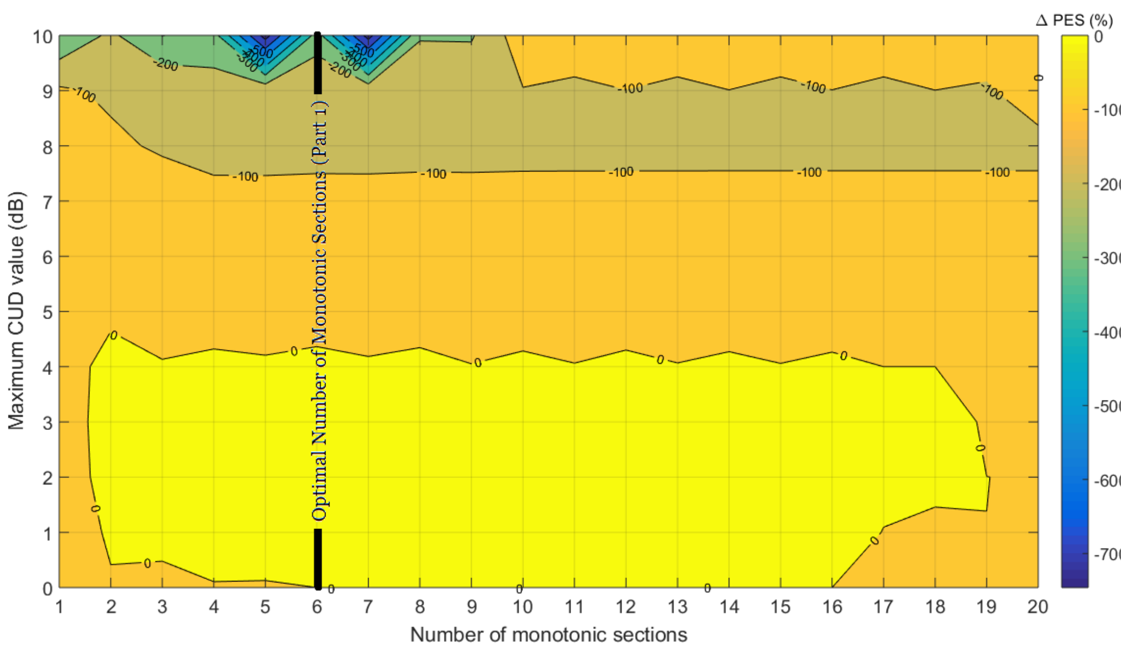


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
Optimal Number of Monotonic Sections (Part 1)

(f)



(g)



(h)

Figure 3. Same with Fig. 1 but for the WtG^3 coupling scheme.

From Figs. 1(a)-(h), 2(a)-(h) and 3(a)-(h), several interesting remarks can be pointed out:

- ΔPES presents significant fluctuations in the 24 cases examined in the aforementioned figures. In fact, its values approximately range from -1600% to 180%. However, positive ΔPES values may occur in all the cases examined indicating that there is always a careful selection of the number of monotonic sections that can mitigate possible measurement differences.

- L1PMA and L2WPMA present significant ΔPES differences even if the same OV MV BPL topology is examined, namely:
    - In the rural and "LOS" cases, L1PMA presents significantly better ΔPES results in comparison with L2WPMA ones regardless of the applied WtG coupling scheme and maximum CUD value. ΔPES differences between two piecewise monotonic data approximations may reach up to 180%. Also, analyzing the morphology of the isodynamic regions, the nature of the approximation procedure remains different between the two investigated approximations in the rural and "LOS" cases; L1PMA best approximates the data by requiring low number of monotonic sections while L2WPMA steadily approximates the same data regardless of the number of monotonic sections.
    - In urban and suburban cases, ΔPES results of L1PMA marginally exceed the respective ones of L2WPMA. Actually, the ΔPES differences between the application of L1PMA and L2WPMA may reach up to 15%. Here, both the applied methods approximate the coupling transfer function data by requiring a number of monotonic sections that ranges between a lower and an upper limit. Below the lower limit, L1PMA and L2WPMA are unable to approximate the rich multipath environments of urban and suburban cases whereas above the upper limit, L1PMA and L2WPMA cannot distinguish spectral notches from severe measurement differences due to the overfit.
- For given piecewise monotonic data approximation and OV MV BPL topology, ΔPES results remain almost the same regardless of the applied WtG coupling scheme. This is a rational event since the coupling transfer function differences among the supported WtG coupling schemes remain small due to the strong presence of the common mode [8], [14], [34].
- The definition of the optimal number of monotonic sections that has been described in [10], [21]-[23] provides an average approximation whether the maximum CUD value is known or not. As described in [10], the optimal number of monotonic sections for the indicative urban, suburban, rural, and "LOS" topologies is equal to 12, 20, 6, and 6, respectively. The line of the optimal number of monotonic sections of [10] is plotted in each figure while it runs through the areas of best ΔPES values in the vast majority of the cases examined either for L1PMA or L2WPMA.
- Also, in the vast majority of the cases examined, L1PMA can better approximate OV MV BPL coupling transfer function data than L2WPMA. This is a result that comes from: (i) the comparison of the respective colorbars next to plots; (ii) the comparison of the optimal ΔPES areas in figures; and (iii) the ΔPES results of the optimal number of monotonic sections. Hence, the main interest of the Sec. IIIB, which deals with the concept of an adaptive behavior against the measurement differences, is focused on L1PMA.

Although the improvement margins for the ΔPES values of L1PMA remain low in urban and suburban topologies when the optimal number of monotonic sections of [10] is adopted, the respective margins in rural and "LOS" cases can become significant. These latter improvements can allow L1PMA to become competitive against L2CXCV in rural and "LOS" cases where piecewise monotonic data approximations with monotonic sections present certain approximation difficulties, which have been described in [10]. The solution of fully exploiting L1PMA potential passes through the proposal of an adaptive number of monotonic sections by taking under consideration an estimation of the maximum CUD value of the surrounding OV MV BPL network environment.

3.2 L1PMA with Adaptive Number of Monotonic Sections versus L1PMA with Optimal Number of Monotonic Sections and L2CXCV

If an accurate estimation of the maximum CUD value can be provided, then the potential ΔPES improvements of L1PMA through the adaptive number of monotonic sections can be significant. In fact, the adaptive number of monotonic sections comes from the localization of the best ΔPES value from the Figs. 1(a)-(h), 2(a)-(h), and 3(a)-(h) given the maximum CUD value, the examined OV MV BPL topology and the applied WtG coupling scheme. Note that greater ΔPES values imply that smaller PES can also be achieved.

In order to demonstrate the improvement potential, L1PMA ΔPES is reported in Table 1 when the optimal number of monotonic sections of [10] and the proposed adaptive number of monotonic sections are applied. To compare the performance improvement, L2WPMA ΔPES and L2CXCV ΔPES are also presented. Finally, In Table 1, all the indicative OV MV BPL topologies are examined when the WtG[1] coupling scheme is applied.

TABLE 1
ΔPES of L1PMA with Optimal Number of Monotonic Sections, L1PMA with Adaptive Number of Monotonic Sections, L2WPMA and L2CXCV for the Indicative Urban OV MV BPL Topology when Different Maximum CUD Values Are Applied
(Blue Font: The Best ΔPES Among the Different Piecewise Monotonic Data Approximations for Given Maximum CUD Value)

Indicative OV MV BPL Topology	Maximum CUD Value	L1PMA / Optimal Number of Monotonic Sections [10]		L2WPMA		L2CXCV	L1PMA / Adaptive Number of Monotonic Sections	
		Number of Monotonic Sections	ΔPES (%)	Number of Monotonic Sections	ΔPES (%)	ΔPES (%)	Number of Monotonic Sections	ΔPES (%)
Urban	0	12	-9.25×10^{-6}	12	-9.70×10^{-6}	-34.55	12 – 20	-9.25×10^{-6}
	1	12	-2.01×10^{-6}	12	-2.13×10^{-6}	-31.13	8 – 9	1.29×10^{-1}
	2	12	-1.08×10^{-1}	12	0.48	-28.41	10 – 11	3.20×10^{-1}
	3	12	-1.65×10^{-1}	12	0.24	-25.72	8 – 9	7.58×10^{-1}
	4	12	-5.93×10^{-1}	12	0.33	-22.32	10 – 11	2.79×10^{-1}
	5	12	5.53×10^{-1}	12	-0.02	-20.30	8 – 9	9.44×10^{-1}
	6	12	8.77×10^{-2}	12	0.81	-15.74	8 – 9	1.71
	7	12	5.15×10^{-2}	12	-5.43×10^{-4}	-17.83	8 – 9	1.31
	8	12	1.06×10^{-1}	12	0.03	-12.02	8 – 9	7.36×10^{-1}
	9	12	1.59×10^{-1}	12	0.16	-10.35	8 – 9	4.99×10^{-1}
	10	12	1.53	12	-0.72	-5.39	4	7.75
Suburban	0	20	-5.36×10^{-6}	20	-5.65×10^{-6}	-36.64	20	-5.36×10^{-6}
	1	20	2.36×10^{-6}	20	1.85×10^{-6}	-31.03	16 – 20	2.36×10^{-6}

	2	20	-5.39×10^-6	20	-4.44×10^-6	-24.43	6 – 7	3.09
	3	20	2.60×10^-6	20	1.34×10^-6	-20.47	18 – 19	1.33×10^-1
	4	20	-1.39×10^-6	20	-2.04×10^-6	-15.77	4	2.57
	5	20	4.96×10^-2	20	1.48	-11.66	10	3.11
	6	20	4.48×10^-6	20	4.93×10^-6	-5.33	4	4.90
	7	20	2.50×10^-6	20	1.87×10^-6	-7.00	4	9.06
	8	20	-4.01×10^-6	20	-2.64×10^-6	0.61	4	7.24
	9	20	-7.02×10^-6	20	6.10×10^-6	4.05	4 – 5	4.75
	10	20	-6.34×10^-6	20	-6.27×10^-6	0.80	1	10.14
Rural	0	6	-5.40×10^-6	6	-5.16×10^-6	-9.45	6 – 20	-5.40×10^-6
	1	6	2.72	6	5.02	2.50	6	2.72
	2	6	5.44	6	6.99	11.74	2	11.41
	3	6	4.36	6	8.87	12.03	2	20.86
	4	6	3.78	6	13.28	16.66	2	28.40
	5	6	11.43	6	-2.53	29.77	1	28.83
	6	6	18.68	6	-6.71	39.46	1	47.26
	7	6	7.39	6	-19.85	46.39	2	55.65
	8	6	13.14	6	-88.57	48.59	1	68.01
	9	6	7.82	6	-75.91	48.79	1	68.18
	10	6	-1.33	6	-155.84	68.45	1	106.96
"LOS"	0	6	-5.18×10^-6	6	-5.24×10^-6	-11.96	6 – 20	-5.18×10^-6
	1	6	3.91	6	5.83	2.12	6	3.91
	2	6	7.38	6	6.62	12.51	2	12.81
	3	6	4.82	6	9.88	12.90	2	23.66
	4	6	4.02	6	8.00	18.06	2	33.57
	5	6	12.88	6	-14.02	32.82	2	39.35
	6	6	22.26	6	-25.05	43.70	1	52.55
	7	6	7.69	6	-33.76	51.10	2	61.84
	8	6	14.27	6	-167.07	53.55	1	73.11
	9	6	8.64	6	-107.21	54.11	1	80.72
	10	6	-1.15	6	-252.97	76.01	1	122.29

From Table 1, it is evident that L1PMA with the adaptive number of monotonic sections achieves better ΔPES against either L1PMA with the optimal number of monotonic sections or L2WPMA or L2CXCV in 38 of 44 cases examined while ΔPES improvement exceeds 15% in the majority of the cases reaching up to 46.28% from the next best approximation.

In addition, it is clearly indicated that even an average estimation of the maximum CUD value can allow better L1PMA approximations. For example, in suburban case, if the occurred maximum CUD value is equal to 6dB and an estimation of the maximum CUD value ranges from 4 to 8dB, the application of less than 10 monotonic sections achieves better ΔPES values than the application of 20 monotonic sections.

Besides, the adaptive number of monotonic sections decreases as the maximum CUD value increases for given OV MV BPL topology and coupling scheme. This is a reasonable result, since the need for a more general approximation is required as the measurement differences start to create great deviations between the theoretical and measured coupling scheme transfer function data. Anyway, this is the main reason for the approximation success of L2CXCV in rural and "LOS" case where the CUD measurement differences are added around quasi-steady OV MV BPL coupling transfer function lines.

Here, it should be noted that the adaptive number of monotonic sections depends not only on the maximum CUD value but on the CUD itself. It is expected that the adaptive number of monotonic sections will little change when maximum values of other CUDs remain below 5dB. However, the robustness of the concept of the adaptive number of monotonic sections against different CUDs of the same maximum CUD value is investigated in the following Sec.IIIC.

Finally, recognizing the previous decreasing trend of the number of monotonic sections with the respective increase of maximum CUD value, this observation may also improve the performance of L1PMA and L2WPMA when the optimal number of monotonic sections is adopted [21]-[23]. Actually, during the selection of the optimal number of monotonic sections when maximum CUD value is assumed equal to 0, the optimal number of monotonic sections is equal to the minimum value of the values that achieve the same ΔPES. For example, in urban, rural, and "LOS" cases, if the optimal number of monotonic sections was assumed equal to 20, 20, and 20, respectively, then ΔPES would be worse for the other maximum CUD values. This comes from the observation that the latter values are far from the respective values of 12, 6, and 6 that are closer to the adaptive number of monotonic sections. Anyway, this remark has already been adopted in [21]-[23].

3.3 L1PMA with Adaptive Number of Monotonic Sections for Different CUDs

The application of L1PMA with the adaptive number of monotonic sections has been examined for a specific set of CUDs in Sec. IIIB. In order to assess the ΔPES performance of L1PMA with the adaptive number of monotonic sections and to generalize the utility value of the adaptive number of monotonic sections shown in Table 1, at least one different set of CUDs with maximum CUD values ranging from 0 to 10dB should be examined.

In order to examine the performance of the adaptive number of monotonic sections, L1PMA ΔPES is reported in Table 2 when the adaptive numbers of monotonic sections of Table 1 are applied. Also, ΔPES of L1PMA with optimal number of monotonic sections, L2WPMA ΔPES and L2CXCV ΔPES are also presented. Furthermore, in Table 2, all the indicative OV MV BPL topologies are examined when the WtG[1] coupling scheme is applied and a different set of CUDs is available.

TABLE 2
Same as Table1 but when the L1PMA Adaptive Number of Monotonic Sections of Table 1 and Different CUD Set Are Assumed
(Blue Font: The Best ΔPES Among the Different Piecewise Monotonic Data Approximations for Given Maximum CUD Value)

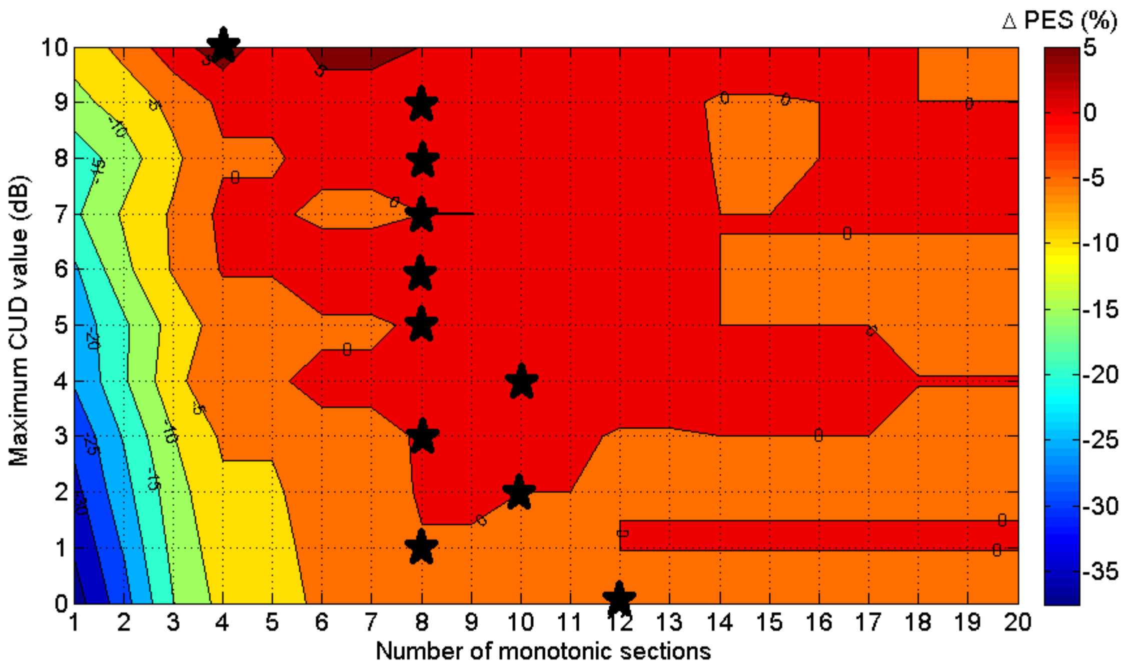
Indicative OV MV BPL Topology	Maximum CUD Value	L1PMA / Optimal Number of Monotonic Sections [10]		L2WPMA		L2CXCV	L1PMA / Adaptive Number of Monotonic Sections	
		Number of Monotonic Sections	ΔPES (%)	Number of Monotonic Sections	ΔPES (%)	ΔPES (%)	Number of Monotonic Sections in accordance with Table 1	ΔPES (%)
Urban	0	12	-9.25×10^{-6}	12	-9.70×10^{-6}	-34.55	12	-9.25×10^{-6}
	1	12	5.92×10^{-7}	12	2.13×10^{-7}	-31.29	8	-2.10×10^{-1}
	2	12	-6.01×10^{-7}	12	-1.08×10^{-6}	-27.21	10	-6.01×10^{-7}
	3	12	-0.11	12	0.002	-23.96	8	3.84
	4	12	0.59	12	1.09	-20.43	10	0.23
	5	12	0.02	12	0.02	-18.92	8	0.85
	6	12	1.05	12	0.95	-17.14	8	3.21
	7	12	0.41	12	-0.40	-9.17	8	-4.71×10^{-2}
	8	12	0.44	12	0.86	-12.93	8	3.02
	9	12	0.36	12	-0.29	-5.13	8	1.58
	10	12	2.02	12	-2.72	-4.60	4	7.76
Suburban	0	20	-5.37×10^{-6}	20	-5.65×10^{-6}	-36.64	20	-5.37×10^{-6}
	1	20	2.67×10^{-6}	20	1.90×10^{-6}	-30.31	16	1.61×10^{-1}
	2	20	1.15×10^{-6}	20	1.39×10^{-6}	-24.02	6	-1.81
	3	20	-1.62×10^{-6}	20	-2.90×10^{-6}	-17.43	18	-1.62×10^{-6}
	4	20	-5.35×10^{-6}	20	-5.11×10^{-6}	-14.19	4	1.54×10^{-1}
	5	20	0.09	20	0.0092	-10.80	10	5.29×10^{-1}
	6	20	3.39×10^{-6}	20	4.21×10^{-6}	-7.40	4	6.74
	7	20	-8.42×10^{-6}	20	-7.92×10^{-6}	-1.29	4	3.78
	8	20	3.44×10^{-6}	20	4.20×10^{-6}	-14.22	4	4.84
	9	20	3.65×10^{-6}	20	3.59×10^{-6}	15.99	4	11.75
	10	20	-7.73×10^{-7}	20	-9.60×10^{-7}	12.75	1	14.01
Rural	0	6	-5.40×10^{-6}	6	-5.16×10^{-6}	-9.45	6	-5.40×10^{-6}
	1	6	1.80	6	4.61	2.52	6	1.80
	2	6	1.12	6	14.68	10.36	2	7.67
	3	6	8.34	6	8.31	24.86	2	22.33
	4	6	14.90	6	6.18	36.42	2	41.63
	5	6	0.10	6	-12.16	33.26	1	36.76
	6	6	13.91	6	-19.70	46.90	1	53.02
	7	6	-3.99	6	-47.91	37.42	2	4.55
	8	6	19.46	6	0.95	59.28	1	54.27
	9	6	13.62	6	-56.98	81.93	1	61.88

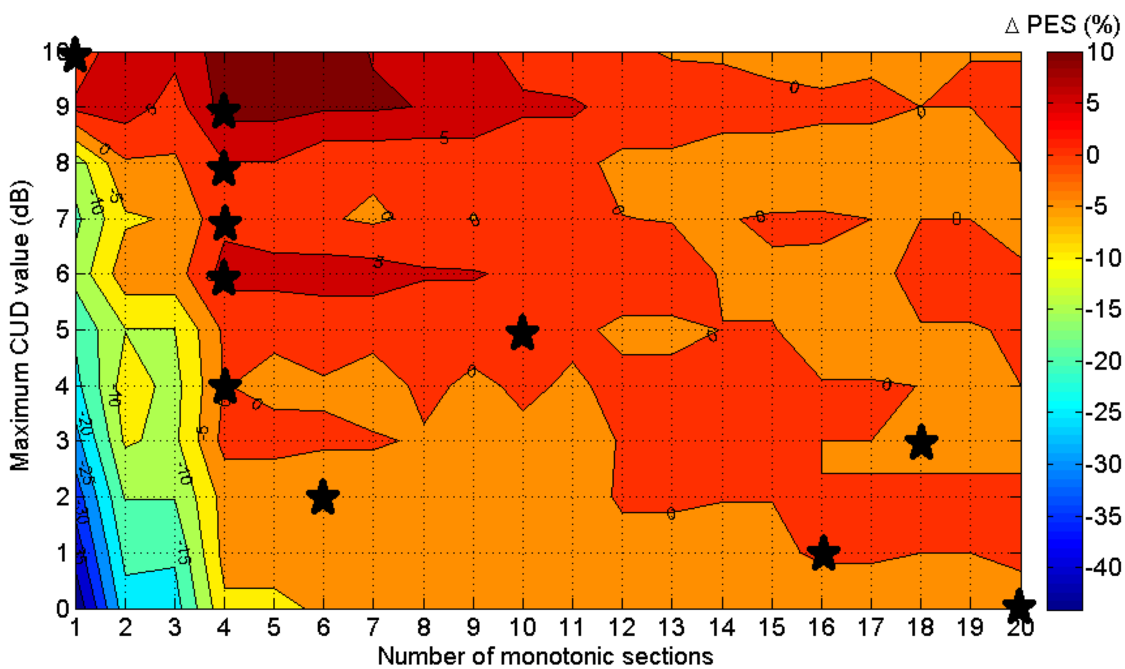
	10	6	18.72	6	-478.80	63.82	1	62.07
“LOS”	0	6	-5.19×10^{-6}	6	-5.24×10^{-6}	-11.96	6	-5.19×10^{-6}
	1	6	1.92	6	4.81	2.09	6	1.92
	2	6	1.91	6	15.54	11.49	2	8.73
	3	6	8.78	6	5.87	27.21	2	24.89
	4	6	17.95	6	-1.73	40.70	2	47.23
	5	6	-0.21	6	0.10	36.94	2	9.44
	6	6	14.53	6	-43.35	48.60	1	57.51
	7	6	-4.73	6	-93.50	41.57	2	2.51
	8	6	21.35	6	-14.51	65.35	1	62.47
	9	6	15.02	6	-122.39	91.15	1	67.07
	10	6	20.23	6	-178.44	70.56	1	66.92

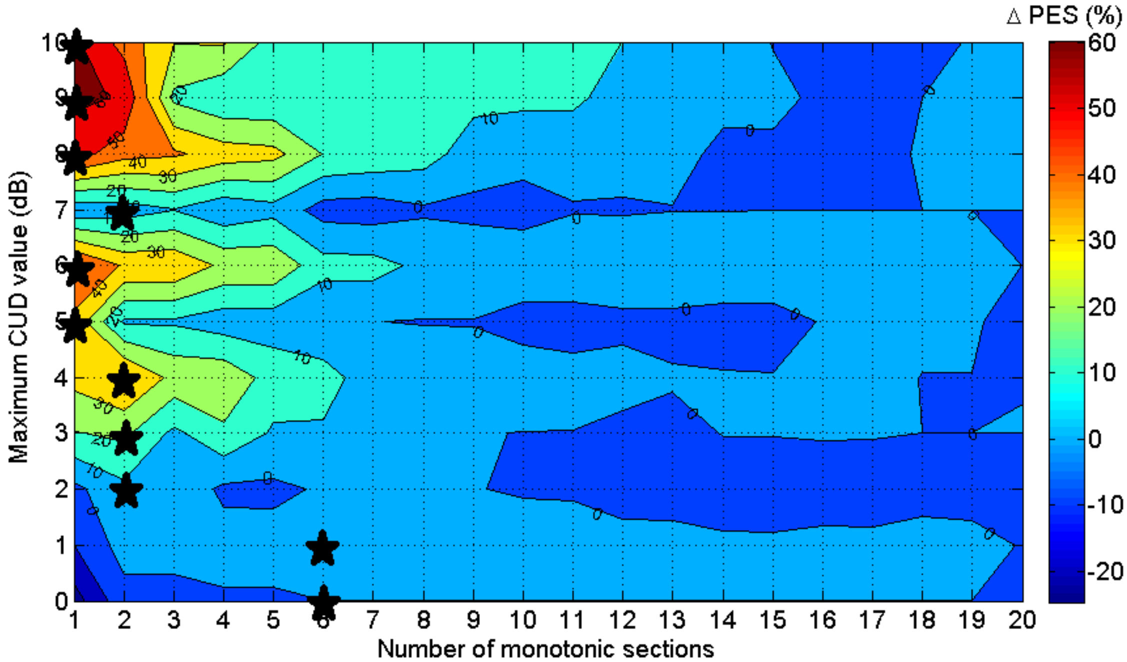
Comparing the ΔPES results of Table 2 with the respective ones of Table 1 and PES results of [10], several general thoughts concerning the concept of the adaptive number of monotonic sections can be expressed:

- It is clearly shown that the performance of L1PMA has significantly been enhanced since the adoption of the concept of the adaptive number of monotonic sections. Indeed, L1PMA with adaptive number of monotonic sections presents better ΔPES results in comparison with the respective ones of L1PMA with the optimal number of monotonic sections even though an arbitrary set of CUDs has been chosen for the evaluation. The performance increase of L1PMA with adaptive number of monotonic sections is based on the need for more general approximations as the maximum CUD value increases and the examined OV MV BPL topology lacks of frequent and short branches across its BPL transmission path.
- Comparing the performance of piecewise monotonic data approximations in Table 1 and 2 with Table 2-5 of [10], the performance improvement of L1PMA with the adaptive number of monotonic sections is evident. In fact, L1PMA with the adaptive number of monotonic sections achieves to give the best approximations in urban and suburban cases while manages to enhance its approximation efficiency in rural and “LOS” cases. Especially, in the case of indicative rural and “LOS” BPL topologies, it should be noted that L2CXCV was competing without an opponent when L2WPMA and L1PMA with the optimal number of monotonic sections were applied. Anyway, L2CXCV better mitigates measurement differences of rural and “LOS” cases when maximum CUD value exceeds 6dB.

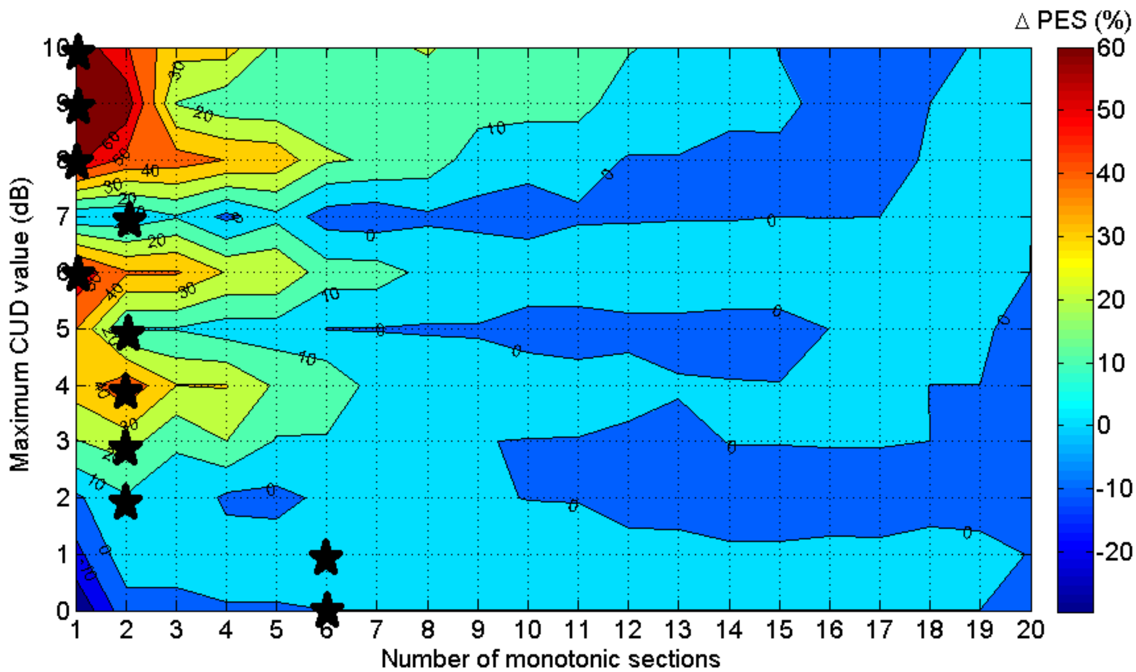
Schematically, the highest mitigating performance of L1PMA with the adaptive number of monotonic sections is presented in Figs. 4(a)-(d) where ΔPES is plotted versus the maximum CUD value and the number of monotonic sections for the indicative urban, suburban, rural and “LOS” case, respectively. In these figures, WtG[1] coupling scheme is as well as the optimal number of monotonic sections in each case is drawn as black stars in accordance with Table 2.


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
(a)


Δ PES (%)
Maximum CUD value (dB)
Number of monotonic sections
(b)



(c)



(d)

Figure 4. ΔPES is plotted for various indicative OV MV BPL topologies with respect to the maximum CUD value and number of monotonic sections for the CUD of Table 2 (WtG[1] coupling scheme is applied). The black stars indicate the adaptive number of monotonic sections for given OV MV BPL topology and maximum CUD value. (a) Urban case / L1PMA. (b) Suburban case / L1PMA. (c) Rural case / L1PMA. (d) "LOS" case / L1PMA.

The improved ΔPES performance of L1PMA with the adaptive number of monotonic sections is also illustrated by the distribution of the number of monotonic sections in Figs. 4(a)-(d). Even though the adaptive number of monotonic sections comes from the numerical analysis of ΔPES results for the set of CUDs of Table 1, the distribution of the adaptive number of monotonic sections, which is illustrated in Figs. 4(a)-(d) with the black stars, is proven to be very accurate for the set of CUDs of Table 2. Indeed, black stars are located in the isodynamic regions of the best ΔPES in the majority of the cases examined. Especially, the adaptive number of monotonic sections in rural and "LOS" cases are in the center of the best ΔPES isodynamic regions, which have significant and frequent changes, exploiting all the mitigating potential of L1PMA against the occurred measurement differences.

Among the future steps of the research concerning the application of piecewise monotonic data approximations in BPL networks, the efficiency of Topology Identification Methodology (TIM) of [22] and Fault and Instability Identification Methodology (FIIM) of [23] needs to be reviewed by taking into account the new findings during the application of L1PMA. Also, other piecewise data approximation methods can be comparatively benchmarked so that better mitigation performance against measurement differences can be established in the future works. Finally, further research and new solutions regarding L1PMA application need to be released in order to explain the better L2CXCV mitigation behavior in rural and "LOS" cases when maximum CUD values exceed 6dB.

Conclusions

In this companion paper, L1PMA efficiency against the measurement differences, which occur during the determination of OV MV BPL coupling transfer functions, has significantly been improved through the adoption of the adaptive number of monotonic sections concept. The behavior of measurement differences has been modeled by continuous uniform distributions (CUDs) with various maximum CUD values.

Although the adaptive number of monotonic sections had been defined in an initial arbitrary CUD set with maximum CUD values ranging from 0 to 10dB, the L1PMA mitigation performance against measurement differences has critically been enhanced even though a different arbitrary CUD set has been chosen for the ΔPES evaluation. In fact, the concept of the adaptive number of monotonic sections is based on the need for more general approximations as the maximum CUD value increases and the examined OV MV BPL topologies lack of frequent and short branches.

The comparative ΔPES benchmark among L1PMA with adaptive number of monotonic sections, L1PMA with optimal number of monotonic sections, L2WPMA with optimal number of monotonic sections and L2CXCV has revealed that the concept of the adaptive number of monotonic sections improves the overall L1PMA performance as follows: (i) L1PMA achieves the best measurement difference mitigation in urban and suburban case without competition. (ii) L1PMA dynamically deals with the measurement differences in rural and "LOS" case when their maximum CUD values remain below 6dB. (iii) Although the performance difference between L1PMA and L2CXCV has drastically been reduced by the use of the adaptive number of monotonic sections, L2CXCV still better approximates rural and "LOS" cases when maximum CUD values exceed 6dB.

Finally, the more effective identification and restoration of the measurement differences during the OV MV BPL coupling transfer function determination may significantly help towards a more stable and self-healing power system.

Conflicts of Interest

The author declares that there is no conflict of interests regarding the publication of this paper.

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