

Footprint Analysis of the Gaussian Mixture Model (GMM) for Populating Overhead Low Voltage Broadband over Power Lines (OV LV BPL) Topology Class Maps

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Until now, Neural Network Topology Generator Methodology (NNTGM) has been theoretically proposed and assessed so that its generated overhead low voltage broadband over power lines topologies (NNTGM OV LV BPL topologies) may populate the existing OV LV BPL topology classes in different footprint patterns on the OV LV BPL topology class maps. With reference to the OV LV BPL topology class maps, which are defined by the graphical combination of average theoretical channel attenuation (ACA) and the standard deviation of the theoretical channel attenuation (SDCA) of the OV LV BPL topologies, Gaussian Mixture Model (GMM) is here proposed and treated as an alternative yet simple population method. In fact, GMM is a weighted sum of multiple Normal Distributions (NDs), which have been until now exploited as error distributions for describing the measurement differences during the BPL topology channel attenuation analysis. In this paper, the effect of the parameters of the GMM on the relative position and the size of the GMM OV LV BPL topology footprints on the OV LV BPL topology class maps is here examined. The effect study of the GMM population is supported by the suitable Graphical Performance Indicators (GPIs) already used during the NNTGM population.

Keywords: Smart Grid; Broadband over Power Lines (BPL) networks; Power Line Communications (PLC); Distribution and Transmission Power Grids; Simulation; Modeling

Introduction

As optical fiber and satellite Internet technologies increasingly dominate global broadband infrastructure, Overhead Low Voltage (OV LV) Broadband over Power Lines (BPL) is emerging as a complementary last-mile communication solution that exploits existing electrical networks to extend connectivity where conventional deployments remain costly or impractical. In fact, the strategic role of OV LV BPL networks could be evolving toward complementing these technologies rather than competing with them [1-4]. Although originally designed for power delivery, OV LV lines can support reliable data communication when their topologies are accurately classified and dynamically managed to address impedance variations, attenuation and noise. Integrating BPL with high-capacity fiber and satellite systems enables hybrid architectures that enhance resilience and coverage, particularly in smart grid and rural contexts. Looking ahead, the

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future of BPL lies in its convergence with machine learning, edge computing and software-defined networking, allowing it to evolve into an intelligent, adaptive layer that supports both broadband access and real-time grid management within increasingly digitalized energy ecosystems [5, 6].

The process of populating OV LV BPL topology classes with appropriate OV LV BPL topologies has become a critical task in modern BPL networks due to the increasing complexity and heterogeneity of contemporary power distribution systems. As BPL technology leverages existing LV power infrastructure for high-speed data transmission, accurately classifying and assigning topologies ensures network performance, improved channel estimation, stability and efficient resource allocation. Furthermore, adaptive modulation and coding strategies can be adopted so that the overall data throughput may enhance. Consequently, this task is essential not only for maintaining the technical efficiency of BPL networks but also for supporting their scalability and integration into smart grid environments of hybrid architectures and various communications technologies [7-9].

The Neural Network Topology Generator Methodology (NNTGM), which has been theoretically proposed in [7] and numerically tested in [8, 9], is an AI-based framework designed to generate virtual OV LV BPL topologies (NNTGM virtual OV LV BPL topologies) that preserve the fundamental topological and channel-related characteristics of the indicative topologies of their corresponding OV LV BPL topology classes. By reversing the network identification methodology for the line length approximation (NNIM-LLA) and employing machine learning techniques [10-12], NNTGM produces virtual topologies that exhibit similar channel attenuation behavior, thereby enabling a realistic and systematic expansion of existing topology classes. NNTGM exploits OV LV BPL topology class maps that are the graphical representation at the same plot of the average theoretical channel attenuation (ACA) and the standard deviation of the channel attenuation (SDCA) of OV LV BPL topologies, which hereafter supersedes the previously used RMS-DS term, maintaining consistency with the present analytical framework. The generated topologies by NNTGM are represented on the OV LV BPL topology class maps through NNTGM virtual OV LV BPL topology footprints, which illustrate the relative position and spread of the virtual topologies with respect to their indicative counterparts and the boundaries of their respective topology classes, showing that they may predominantly lie within these boundaries but, depending on the selected parameters and their variations, can also extend beyond them, thus generating footprints of different tendencies and morphologies. Furthermore, various operational parameters of NNTGM were examined —such as the inclusion or not of the examined indicative OV LV BPL topology in the Topology Identification Methodology (TIM) OV LV BPL topology database during the NNTGM operation, the length of the distribution / branch line segments; the number of the distribution / branch line segments, the operating frequencies (i.e., frequency range and flat-fading subchannel frequency spacing) and the participation percentages of the three phases (training, validation and testing) of the MATLAB NN training program of NNTGM— resulting in the formation of distinct virtual topology footprints that reflect the influence of these parameters on the generated topological distributions.

For populating OV LV BPL topology class maps with virtual topology footprints of various relative positions, dispersions, tendencies and morphologies, the Gaussian Mixture Model (GMM) is proposed in this paper. Actually, GMM is a probabilistic model that represents a distribution as a weighted sum of multiple Normal (Gaussian)

Distributions (NDs), where each ND component captures one cluster of the data of the virtual topology footprint. Within this context, a key advantage of employing a GMM lies in its inherent structural simplicity. Each ND component is fully specified by its weight together with its corresponding mean μ and standard deviation σ , thereby enabling direct and interpretable control over the shape, orientation and spatial extent of the resulting GMM virtual OV LV BPL topology footprints on the OV LV BPL topology class maps. Consequently, the modeling process remains transparent, as the final GMM virtual OV LV BPL topology footprint is not the output of a ‘black-box’ mechanism but rather the superposition of simple and well-understood NDs. By comparison, the individual NDs that constitute the GMM are capable of generating footprints with clear geometric structure, as has also been demonstrated by their influence on class maps when examined in the context of measurement differences [13-19]. In contrast, the NNTGM requires the specification of a set of NNTGM operation settings and the execution of the MATLAB NN training program [20, 21], which introduce additional complexity, uncertainty and high-dimensional parameterization. The representation of local virtual topology morphology emerges implicitly through the learning dynamics of the network, making it more difficult to ascertain how input parameters translate into spatial characteristics of the produced NNTGM virtual OV LV BPL topology footprints on the OV LV BPL topology class maps. The use of a GMM therefore enables the formation of more controllable, predictable and analytically interpretable virtual footprints, without the need for complex NN training procedures or extensive tuning of operational settings.

Based on the OV LV BPL topology class maps and the corresponding graphical performance indicators (GPIs) from the NNTGM analysis of [7-9], the impact of the GMM parameters on the relative position and the size of the GMM virtual topology footprints are examined in this paper. To assess the relative position and the size of the GMM virtual topology footprints with respect to the corresponding OV LV BPL topology class borderlines, two GPIs have been applied in [8, 9] and are adopted in this GMM study; say, the percentage of the GMM virtual OV LV BPL topologies that remain inside the OV LV BPL topology class of their respective indicative OV LV BPL topology and the percentage of the GMM virtual OV LV BPL topologies that remain inside specific ACA / SDCA limits with respect to their indicative OV LV BPL topology. In this paper, the impact of the following GMM parameters on the resulting GMM virtual topology footprints and GPIs is further examined, namely: (i) the mean μ of the NDs, (ii) the standard deviation σ of the NDs, (iii) the total count of NDs considered in the application of the GMM, and (iv) the weight assigned to each ND included in the GMM. The findings of this paper support the development of an expanded populating strategy for the OV LV BPL topology class, thereby facilitating a more streamlined and transparent populating process.

The rest of this paper is organized as follows: Section 2 synthesizes the basics of class maps, footprints and GPIs of the indicative OV LV BPL topologies when GMM is going to be applied for populating OV LV BPL topology class maps. Section 3 analyzes the impact of the aforementioned GMM parameters on the GMM virtual topology footprints and GPIs. Section 4 concludes this paper.

GMM Definition, OV LV BPL Topology Class Maps, Virtual Topology Footprints and GPIs

In this Section, the GMM is considered as an alternative modeling framework to the NNTGM. To enable a consistent assessment of the GMM under variations of its parameters, the default GMM operation settings are first presented. Moreover, the channel attenuation metrics —i.e., ACA and SDCA— as well as the OV LV BPL topology class maps, which were originally defined in [7] for the NNTGM case, are adopted here for the analysis of the GMM to ensure methodological consistency and comparability. In addition, for benchmark purposes, these metrics and OV LV BPL topology class maps are provided for the indicative OV LV BPL topologies of [7] together with their corresponding GMM virtual OV LV BPL topologies. Finally, the GPIs, initially introduced in [8] and used to evaluate the impact of changes of the GMM parameters on the relative position and size of the GMM virtual topology footprints on the OV LV BPL topology classes, are also included in this Section.

GMM Definition and GMM Operation Settings

GMM builds on the prior use of NDs solely as statistical models of measurement differences [13-19], extending this earlier experience by employing these ND-derived characterizations within a more structured framework for populating OV LV BPL topology classes and generating GMM virtual OV LV BPL topologies. Actually, a set of practical reasons and “real life” conditions, which can be grouped into six categories [13, 18, 19], can create measurement differences during the practical determination of the coupling scheme transfer function of the OV LV BPL topologies. Appropriately modifying eq. (1) of [14], the measured coupling scheme transfer function, which is treated as the virtual coupling scheme transfer function $H_{virtual}(f_q)$, $q = 1, \dots, Q$ in this paper, is obtained by superimposing on the theoretical coupling scheme transfer function $H_{theor}(f_q)$, $q = 1, \dots, Q$, which is produced by the Deterministic Hybrid Model (DHM), a synthetic BPL channel model composed of three concatenated modules, namely the bottom-up, top-down and coupling-scheme modules [22, 23], the measurement differences in dB that are described by the random variable $e^D(f_q)$, $q = 1, \dots, Q$, namely:

$$H_{virtual}(f_q) = H_{theor}(f_q) - e^D(f_q), q=1,\dots,Q \quad (1)$$

where $[\cdot]^D$ denotes the applied difference distribution —i.e., either Continuous Uniform Distribution (CUD) or ND—, f_q is the flat-fading subchannel spacing and the start frequency in this paper, and Q is the number of subchannels in the examined frequency range. By employing this methodology, which originally generated measurement differences using simple CUDs or NDs, and now replacing these with a GMM composed of NDs with varying means, standard deviations and weights, it becomes possible to generate virtual OV LV BPL topologies with diverse characteristics, depending on the selected values of the aforementioned GMM parameters. By combining multiple NDs, GMM can capture a wide range of variability and complexity that anyway characterize the topological characteristics of OV LV BPL topologies. Therefore, GMM virtual OV LV BPL topologies are expected to present channel attenuation behaviors that can remain close to the one of the examined indicative OV LV BPL topology in each case but this can be further adjusted through the change of the parameters of the GMM operation settings as it has been shown in the case of NNTGM in [8].

As GMM is concerned, it is employed to model the distribution of the vector $\mathbf{e}^{\text{GMM}} = [e^{\text{GMM}}(f_1), \dots, e^{\text{GMM}}(f_Q)]$, which contains the error samples across all examined frequencies. The corresponding probability density function (PDF) is expressed as a weighted sum of K ND components:

$$p(\mathbf{e}^{\text{GMM}}) = \sum_{k=1}^K w_k p(\mathbf{e}^{\text{GMM}}|k) \quad (2)$$

where w_k denotes the weight of k -th ND component with the constraint $\sum_{k=1}^K w_k = 1$. Each ND component corresponds to a distinct cluster once a component k is selected according to w_k , the samples $e^{\text{GMM}}(f_q)$ for all frequencies $q = 1, \dots, Q$ are generated from the corresponding ND component parameters $(\mu_k(f_q), \sigma_k(f_q))$. Consequently, the same ND component governs the values of $e^{\text{GMM}}(f_q)$ across the entire frequency range, which naturally leads to the formation of clusters when multiple ND components are included in the GMM.

As the GMM default operation settings are concerned, these can be listed as follows in accordance with the ones of NNTGM of [7, 8]:

- OV LV BPL networks are divided into cascaded OV LV BPL topologies of typical lengths of 1000m while a typical OV LV BPL topology is shown in Figure 1 of [7]. With reference to Figure 1 of [7], the typical OV LV BPL topology is bounded by its transmitting and receiving ends. Across the transmission path of the OV LV BPL topology, N branches of various lengths and of open-circuit terminations may occur. In accordance with [7, 10, 18, 23], the topological characteristics and the number of branches for the three indicative OV LV BPL topologies –i.e., rural, suburban and urban– are listed in Table 1 of [7]. These three indicative OV LV BPL topologies are the representative ones of the respective main OV LV BPL topology classes, which are also detailed in [13-15].
- Line-Of-Sight (LOS) topology case, which is also listed in Table 1 of [7], is characterized by the minimum achievable attenuation within the family of OV LV BPL topologies of interest. Accordingly, the values of eq. (2) produced by the GMM are constrained such that the channel attenuation of the virtual OV LV BPL topology, as given in eq. (1) for a specified frequency, can never be lower than the attenuation corresponding to the LOS topology case. This LOS cutoff constraint guarantees that the statistical modeling remains physically consistent and adheres to the lower-bound attenuation behavior dictated by LOS propagation conditions.
- As far as the operating frequencies of the GMM are concerned, the frequency range is assumed to span 1-30MHz, whereas the flat-fading subchannel frequency spacing is set to 1MHz. Under these assumptions, the total number of flat-fading subchannels Q equals 29. Note that for reasons of methodological consistency and in order to facilitate a direct and meaningful comparison between GMM and NNTGM, the same assumptions regarding the operating frequency range and the flat-fading subchannel spacing in NNTGM are also applied to the GMM. This alignment ensures that differences observed between NNTGM and GMM results can be attributed to the modeling schemes themselves rather than to discrepancies in the underlying frequency-domain parametrization.

Similarly to the NNTGM, the evaluation of the GMM is conducted by examining the channel attenuation metrics ACA and SDCA which are employed in the construction of the OV LV BPL topology class maps of this paper, along with the corresponding GPIs that are presented in the following subsection.

Channel Attenuation Metrics, OV LV BPL Topology Class Maps and GPIs for the GMM Evaluation

Similarly to the class maps of [13, 15, 24-26], OV LV BPL topology class maps of [7-9], which are going to be exploited by GMM case, are plotted by using the channel attenuation metrics ACA and SDCA for the horizontal and vertical axes, respectively. With reference to eqs. (11) and (12) of [7], ACA can be calculated for each OV LV BPL topology (either indicative or NNTGM virtual topology or GMM virtual one) by averaging the amplitudes of the coupling scheme channel attenuations in dB over the flat-fading subchannels while SDCA measures the average channel attenuation distance from ACA for the examined OV LV BPL topologies over the same flat-fading subchannels. Similarly to NNTGM approach of [7-9], by leveraging the combination of ACA and SDCA values, each indicative OV LV BPL topology, along with its corresponding GMM virtual topologies, can be represented graphically to delineate its OV LV BPL topology footprint on the class map. As discussed in [7-9], the footprints of these indicative OV LV BPL topologies exhibit both the origin and relative proximity to their respective topologies, while their specific locations and dimensions on the class map are going to be determined by the applied parameters of the GMM operation settings. To illustrate the OV LV BPL topology class areas on the class maps and to investigate the behavior of the corresponding topology footprints, two graphical milestones are applied. First, the class boundaries are approximately determined as the median lines between neighboring indicative OV LV BPL topologies, as reported in [7-9], under the default NNTGM and GMM operation settings; specifically, these are the rural–suburban and suburban–urban class boundaries. Second, an ACA / SDCA box corresponding to $\pm 10\%$ variation is defined around each indicative OV LV BPL topology, reflecting the effect of a $\pm 10\%$ change in ACA and SDCA values for the applied default NNTGM and GMM operation settings.

With reference to [7-9], in Figure 1(a), the OV LV BPL topology class map with the indicative rural case, suburban case and urban case A is presented when the default NNTGM and GMM operation settings are applied. Apart from the two class borderlines, the three ACA / SDCA boxes of $\pm 10\%$ of the respective three indicative OV LV BPL topologies are illustrated with dashed lines. In Figure 1(b), to enable a direct comparison between the subsequent GMM results and those obtained using the NNTGM, the NNTGM virtual topology footprints for the rural, suburban, and urban case A scenarios of [7-9] are superimposed onto the OV LV BPL topology class map of Figure 1(a). This superposition is performed using the default NNTGM operational settings and includes 100 NNTGM virtual topologies for each corresponding topology footprint.

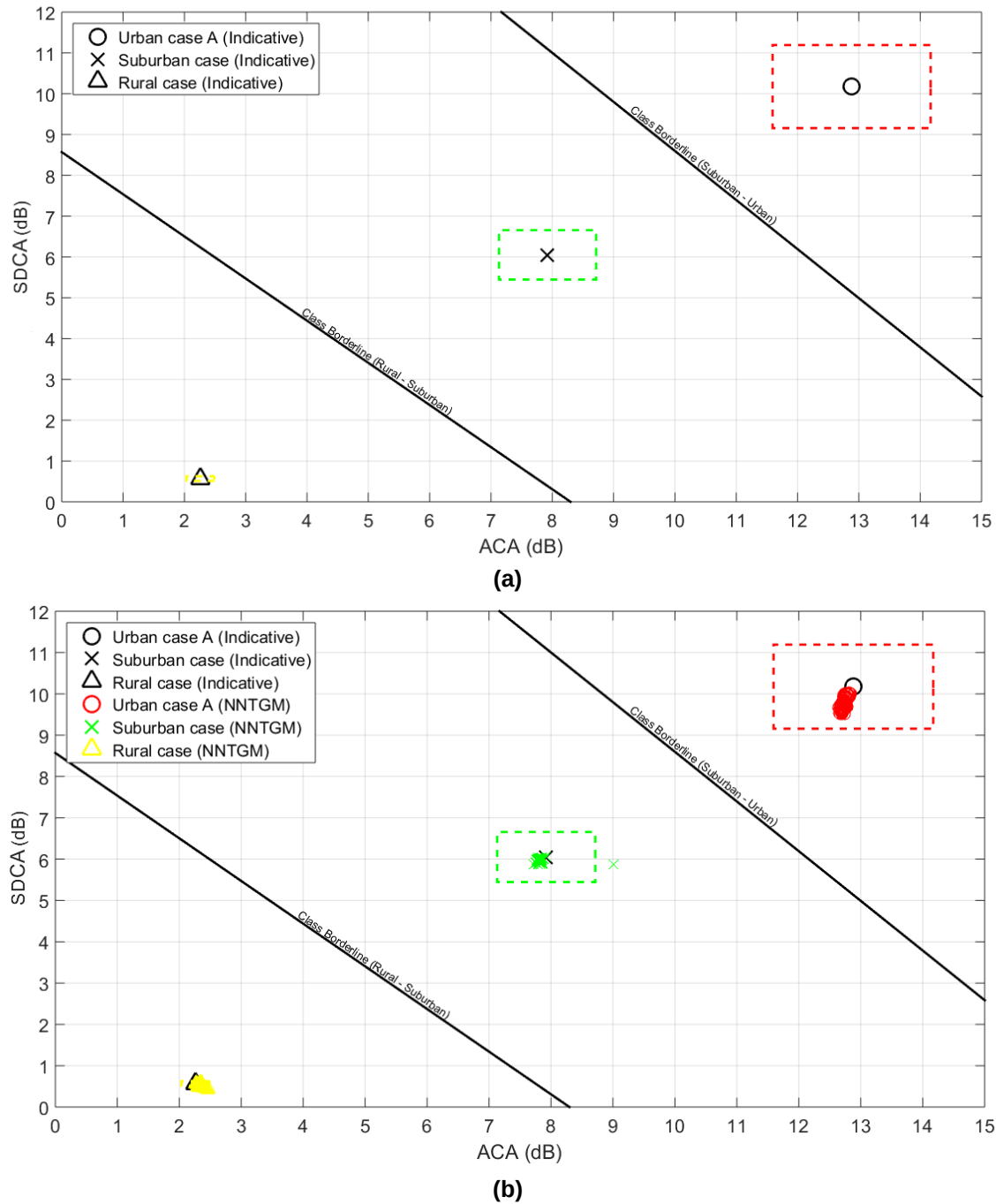


Fig. 1. (a) OV LV BPL topology class map of the three indicative OV LV BPL topologies when the NNTGM and GMM default operation settings are applied. **(b)** OV LV BPL topology class map with NNTGM virtual topology footprints of the three indicative OV LV BPL topologies when the NNTGM default operation settings are applied [7-9].

With reference to the two previously defined graphical milestones, two GPIs have been used in [7-9] for NNTGM and are also going to be applied in GMM to assess the distribution of OV LV BPL topology classes under the applied GMM operation settings, namely:

- *Inside-the-class*: This GPI is going to quantify the percentage of GMM virtual OV LV BPL topologies that remain within the class corresponding to their respective indicative OV LV BPL topology.
- *Inside-the-box*: This GPI is going to measure the percentage of GMM virtual OV LV BPL topologies that remain within a $\pm 10\%$ ACA / SDCA box relative to their indicative OV LV BPL topology.

The combined use of class maps and GPIs provides a robust framework for assessing the influence of GMM parameters on the resulting GMM virtual OV LV BPL topology footprints. To enable a direct GPI-based comparison between the NNTGM and GMM, the aforementioned GPIs that are previously applied to the NNTGM OV LV BPL topologies of the three OV LV BPL topology classes are summarized in Table 1.

The objective of Figure 1(b) and Table 1 is to transform the findings of [7-9] presented for NNTGM into a quantitative reference baseline. This allows the effects of parameter variations applied in GMM, which are examined in the following Section, to be assessed relative to NNTGM results of Figure 1(b) and Table 1, enabling a clear quantitative evaluation of their impact and a precise understanding of how the outcomes shift in relation to the NNTGM baseline.

The Effect of GMM Parameter Variation on the GMM Virtual Topology Footprints

In this Section, the impact of the following GMM parameters on the resulting GMM virtual topology footprints and GPIs is examined, namely: (i) the mean μ of the NDs, (ii) the standard deviation σ of the NDs, (iii) the total count of NDs considered in the application of the GMM, and (iv) the weight assigned to each ND included in the GMM. The results presented in this paper provide a foundation for devising a simplified strategy for populating OV LV BPL topology classes, thereby enabling a more efficient and transparent class population process.

ND Mean and its Impact on GMM Virtual Topology Footprints and GPIs

GMM represents the distribution of the aforementioned random variable $e^{\text{GMM}}(f_q), q = 1, \dots, Q$ of eq. (1) as a weighted sum of K ND components. Each ND component is characterized by its mean μ_k and standard deviation σ_k in order to model a distinct cluster within the data.

As the impact of the ND mean is concerned, in Figure 2(a), the OV LV BPL topology class map with the GMM virtual topology footprints of the rural case, suburban case and urban case A is presented when the GMM default operation settings are applied and GMM consists of only one ND component with ND mean μ and ND standard deviation σ that are equal to 0dB and 0.5dB, respectively. Similarly to Figure 1(b) of NNTGM, each GMM virtual OV LV BPL topology footprint of Figure 2(a) comprises 100 GMM virtual topologies per indicative OV LV BPL topology.

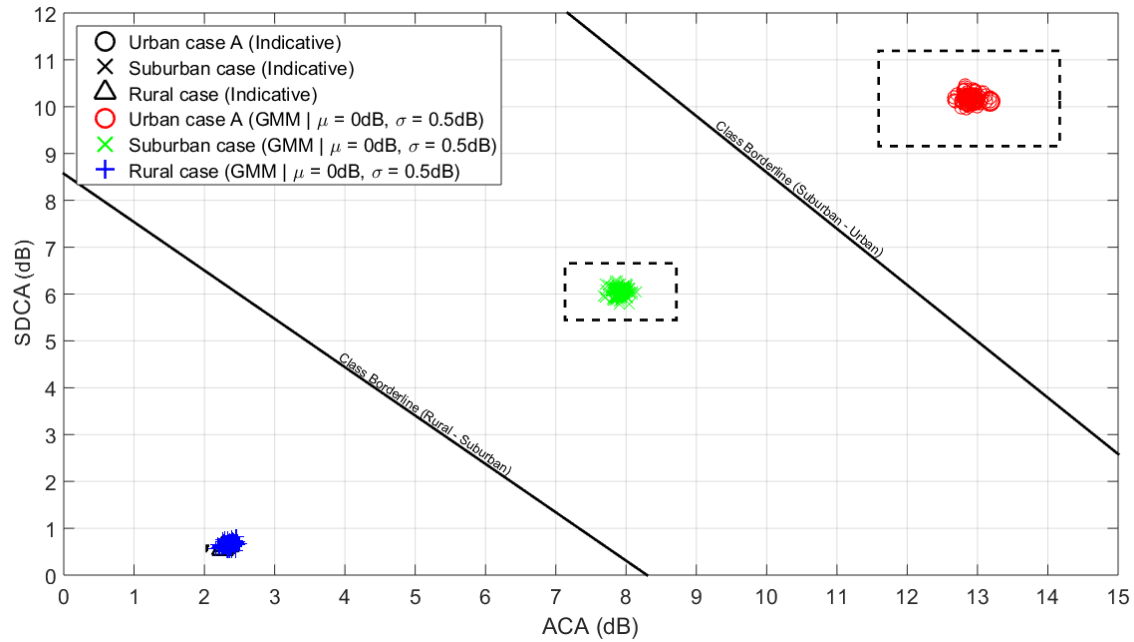
Table 1. GPIs of the NNTGM virtual OV LV BPL topologies of the three OV LV BPL topology classes (NNTGM default operation settings are applied) [8]

GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	84	99	100

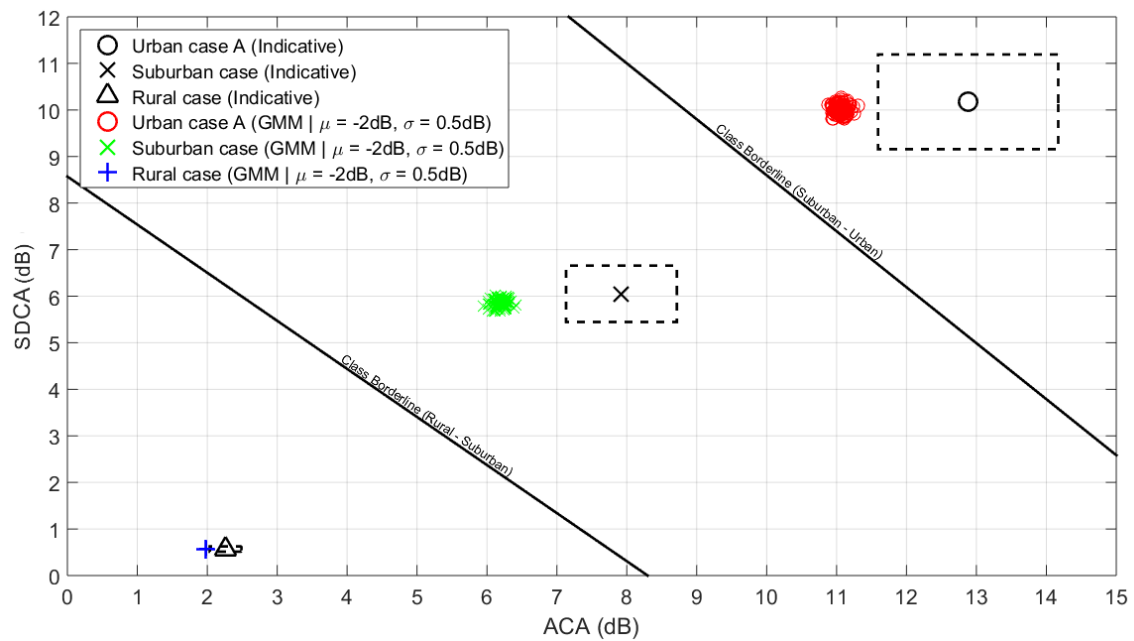
The two class borderlines of Figures 1(a) and 1(b) and the three ACA / SDCA boxes of $\pm 10\%$ of the respective three indicative OV LV BPL topologies are also added. In Figures 2(b)-2(e), similar plots to Figure 2(a) are given, but for the case of ND mean μ of -2dB, +1dB, +3dB and +5.5dB, respectively. The same GPIs of Table 1 are applied to the GMM virtual OV LV BPL topologies of the three OV LV BPL topology classes of Figures 2(a)-2(e) and are reported in Table 2.

Figures 2(a)-2(e) illustrate the GMM virtual OV LV BPL topology footprints corresponding to the rural, suburban and urban A cases under varying values of the ND mean μ , while the ND standard deviation σ remains fixed at 0.5dB. The choice of such a small σ inherently limits the statistical dispersion of the ND values, resulting in compact and concentrated GMM virtual OV LV BPL topology footprints rather than wide or diffuse footprint regions; this reflects the constrained variability of ACA and SDCA around each footprint centroid. As μ increases from -2 dB to +3 dB, all three examined cases exhibit a systematic and approximately proportional rightward displacement of the centroid of the corresponding generated GMM virtual OV LV BPL topology footprint. This trend indicates a consistent shift toward higher ACA levels across all cases, while the SDCA values of the footprints remain nearly unchanged. Consequently, the resulting centroid movement is predominantly horizontal, demonstrating that the GMM components reconfigure themselves in a similar manner across the different topology classes to represent progressively more degraded propagation conditions without modifying the dispersion characteristics of the GMM virtual OV LV BPL topologies. As the footprint of rural case is concerned when $\mu = -2$ dB and σ is small, the LOS cutoff, which has been presented during the definition of the GMM default operation settings, becomes active for the GMM virtual OV LV BPL topologies of the rural case, causing the rural footprint to adhere to the LOS baseline and concentrate against it rather than sit below it; this clipping effect yields a markedly reduced spatial extent for the rural footprint compared with the suburban and urban A cases at the same ND mean value. Overall, the GMM virtual OV LV BPL topology footprints clearly demonstrate how the ND statistical parameters may shape the inferred virtual topologies, highlighting the interaction between environment type, mean channel degradation and the structural behavior of the GMM representation.

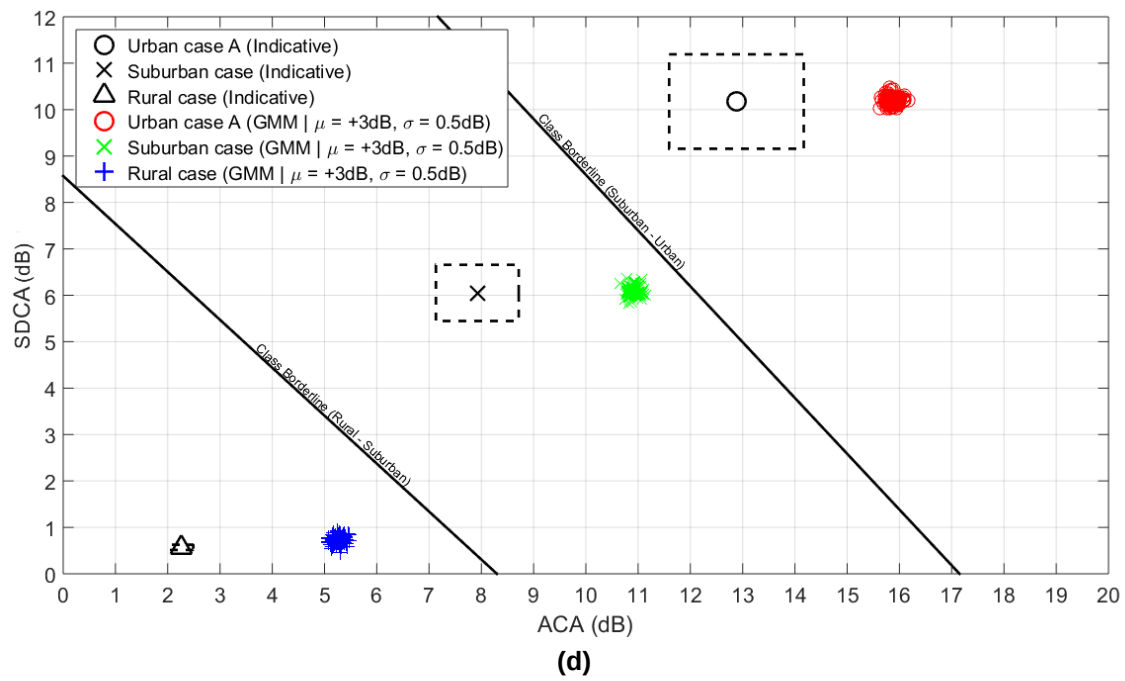
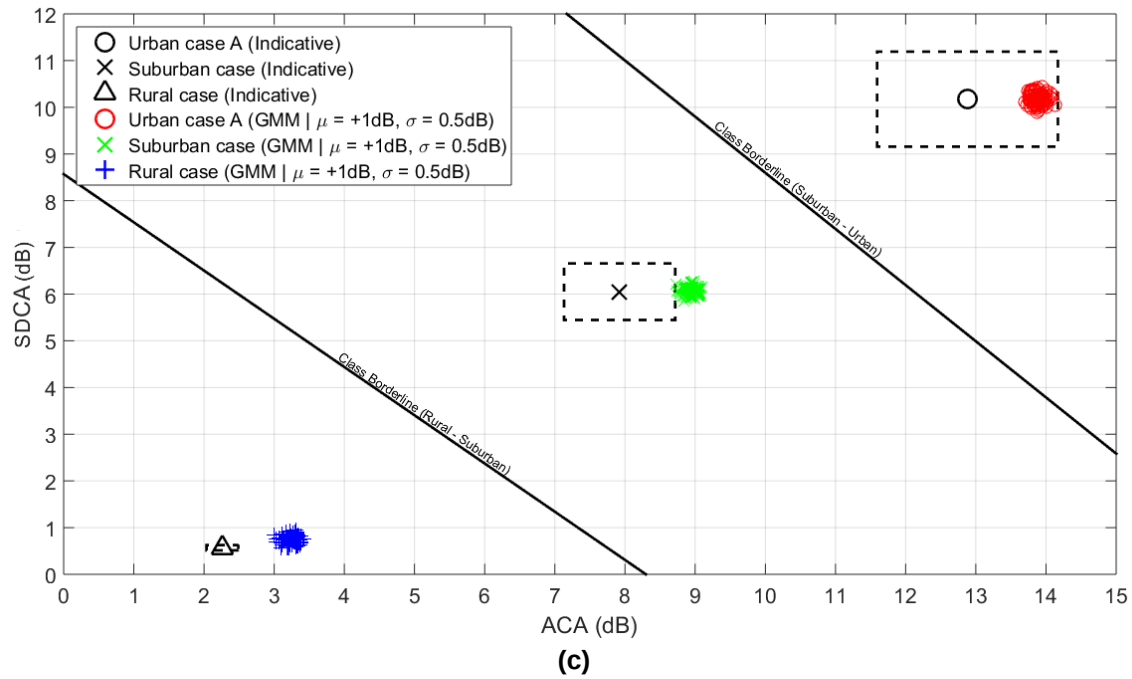
Combining findings of Figures 2(a)-2(e) with Table 2, it is shown that although all footprints remain fully consistent with their corresponding topology class (100% inside-the-class), their ability to fall within the predefined ACA / SDCA bounding box of the indicative OV LV BPL topology of origin is highly sensitive to the ND mean μ . At $\mu = 0$ dB the suburban and urban A footprints fully satisfy the box constraint while the rural footprint only partially does so, whereas for $\mu = -2$ dB and $\mu = +3$ dB all cases fall entirely



(a)



(b)



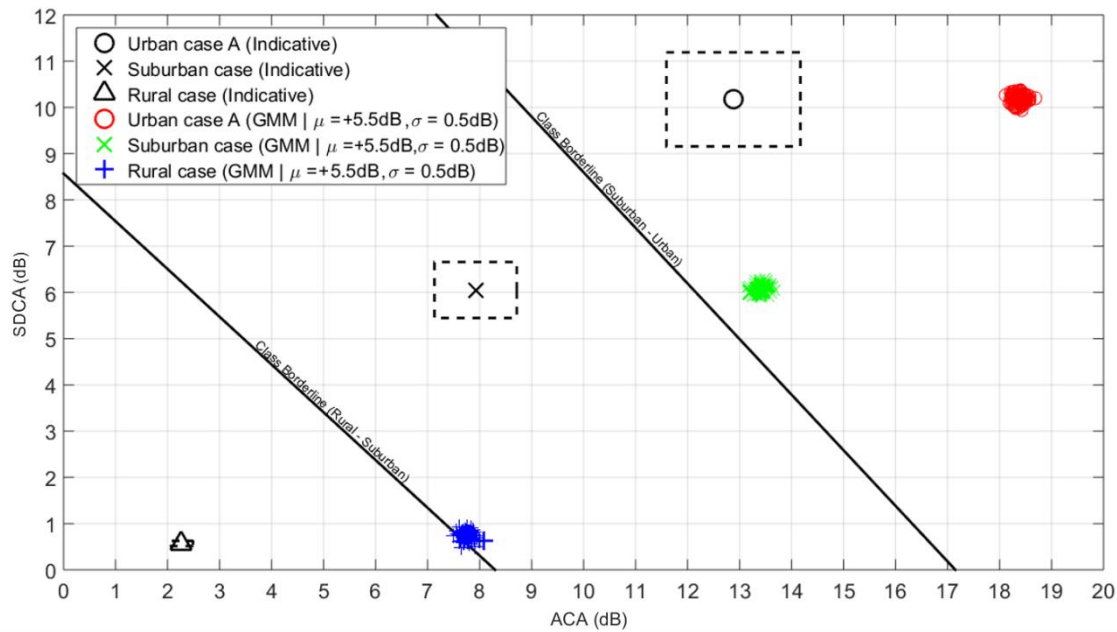


Fig. 2. OV LV BPL topology class map with GMM virtual topology footprints of the three indicative OV LV BPL topologies for various single ND mean values. **(a)** ND mean $\mu = 0\text{dB}$. **(b)** ND mean $\mu = -2\text{dB}$. **(c)** ND mean $\mu = +1\text{dB}$. **(d)** ND mean $\mu = +3\text{dB}$. **(e)** ND mean $\mu = +5.5\text{dB}$.

Table 2. GPIs of the GMM OV LV BPL topologies of the three OV LV BPL topology classes for different ND mean μ values when the ND standard deviation σ is equal to 0.5dB (GMM default operation settings are applied).

ND mean $\mu = 0\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	32	100	100
ND mean $\mu = -2\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	0	0	0
ND mean $\mu = +1\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	0	0	100

ND mean $\mu = +3\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	0	0	0

ND mean $\mu = +5.5\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	3	0	100
Inside the box (%)	0	0	0

outside the box due to the strong leftward and rightward shifts in ACA, respectively. As μ increases, only the urban case A footprint retains full box compliance at $\mu = +1\text{dB}$, while both rural and suburban cases progressively drift outside the bounding region, indicating that higher ND means push these environments into attenuation regimes outside the box constraints. Naturally, the definition of the $\pm 10\%$ ACA / SDCA box also plays an important role. The $\pm 10\%$ margin applied to the box of the rural case provides limited tolerance for variability in the GMM virtual OV LV BPL topologies if they are to remain within its bounds, in contrast to the more relaxed box constructed for the urban case A. Regarding the GPI inside-the-class, regardless of how large the ND mean μ becomes when generating GMM virtual OV LV BPL topologies for the urban case A, the suburban-urban class borderline remains a limiting boundary. Consequently, these topologies always reside within the urban case A class, which justifies the constant value of 100 for the GPI inside-the-class across all entries in Table 2 and the following Tables.

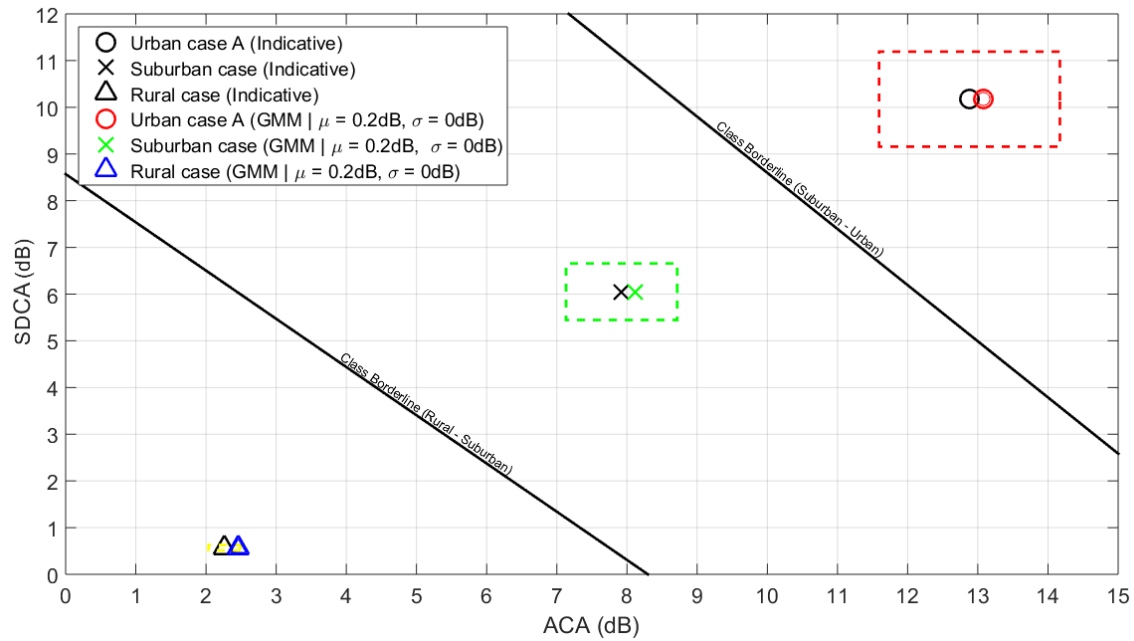
As the impact of the ND mean is examined on the population of OV LV BPL topology classes, higher ND means shift the footprints toward regions of increased ACA, which in turn enables targeted population of areas corresponding to higher-ACA OV LV BPL topology classes even when these regions lie at the boundaries of, or slightly outside, the original classes from which the GMM virtual OV LV BPL topologies are derived. This behavior highlights that the GMM virtual footprints can populate topological zones beyond their nominal class cores as μ increases, maintaining similar SDCA elevations and remaining approximately parallel along the ACA axis. Comparing Figure 1(b) and Table 1 of NNTGM with Figure 2(a) and Table 2 of GMM, respectively, the NNTGM and GMM population approaches show that both methods consistently achieve 100% GPI inside-the-class for the rural, suburban and urban cases, confirming stable class preservation under their respective generation mechanisms. However, their behavior diverges in the GPI inside-the-box, where NNTGM under default operation settings exhibits a gradual transition across cases (84% rural, 99% suburban, 100% urban), while GMM—evaluated at $\mu = 0\text{ dB}$, $\sigma = 0.5\text{ dB}$ —demonstrates a more abrupt, almost step-like response (32% rural, 100% suburban and 100% urban). Anyway, in accordance with [7-9], NNTGM can perform various population schemes for its virtual OV LV BPL topology footprints by appropriately modifying its default operation settings but the further comparison is outside the scope of this paper.

ND Standard Deviation and its Impact on GMM Virtual Topology Footprints and GPIs

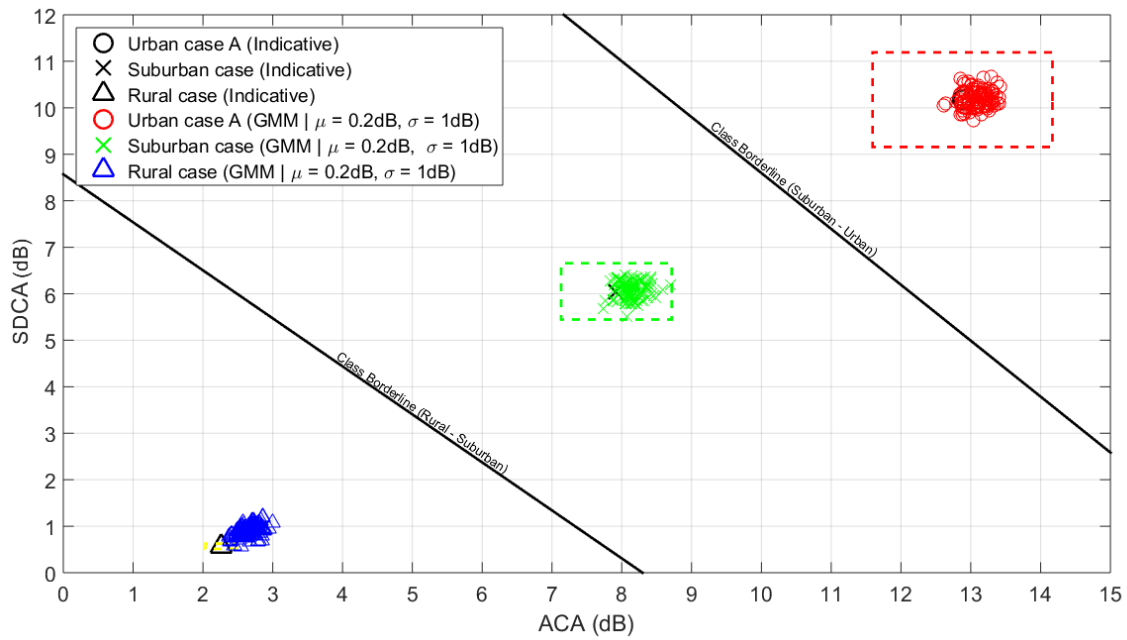
As the impact of the ND standard deviation is concerned, in Figure 3(a), the OV LV BPL topology class map with the GMM virtual topology footprints of the rural case, suburban case and urban case A is presented when the GMM default operation settings are applied and GMM consists of only one ND component with ND mean μ and ND standard deviation σ that are equal to 0.2dB and 0dB, respectively. Each GMM virtual OV LV BPL topology footprint of Figure 3(a) comprises 100 GMM virtual topologies per indicative OV LV BPL topology. In Figures 3(b)-3(e), similar plots to Figure 3(a) are given, but for the case of ND standard deviation σ of 1dB, 3dB, 5dB and 7dB, respectively. The same GPIs of Tables 1 and 2 are applied to the GMM virtual OV LV BPL topologies of the three OV LV BPL topology classes of Figures 3(a)-3(e) and are reported in Table 3.

Figures 3(a)-3(e) illustrate the GMM virtual OV LV BPL topology footprints corresponding to the rural, suburban and urban A cases under varying values of the ND standard deviation σ , while the ND mean μ remains fixed at 0.2dB. The increase of the ND standard deviation σ from 0dB to 7dB results in a rightward displacement of the GMM virtual OV LV BPL topology footprint centroids, not merely parallel to the ACA axis as observed previously in Figures 2(a)-2(e), but following a diagonal trajectory towards the upper-right region of the ACA / SDCA plane, thereby engaging the vertical SDCA dimension more prominently. Furthermore, in contrast to the behavior observed under increasing ND mean, where the spatial spread of the GMM virtual OV LV BPL topology footprints remained compact as long as σ was held small and constant, here the increase in ND standard deviation induces a noticeable spatial expansion of the footprints for all examined indicative OV LV BPL topologies. This diagonal migration is more pronounced in the rural case, where several GMM virtual OV LV BPL topologies have already exceeded the rural-suburban class borderline; a similar trend is present in the suburban case, albeit in a more moderate form. While the urban case A also exhibits detectable rightward movement, the dominant effect is the substantial increase in footprint spatial extent as ND standard deviation grows.

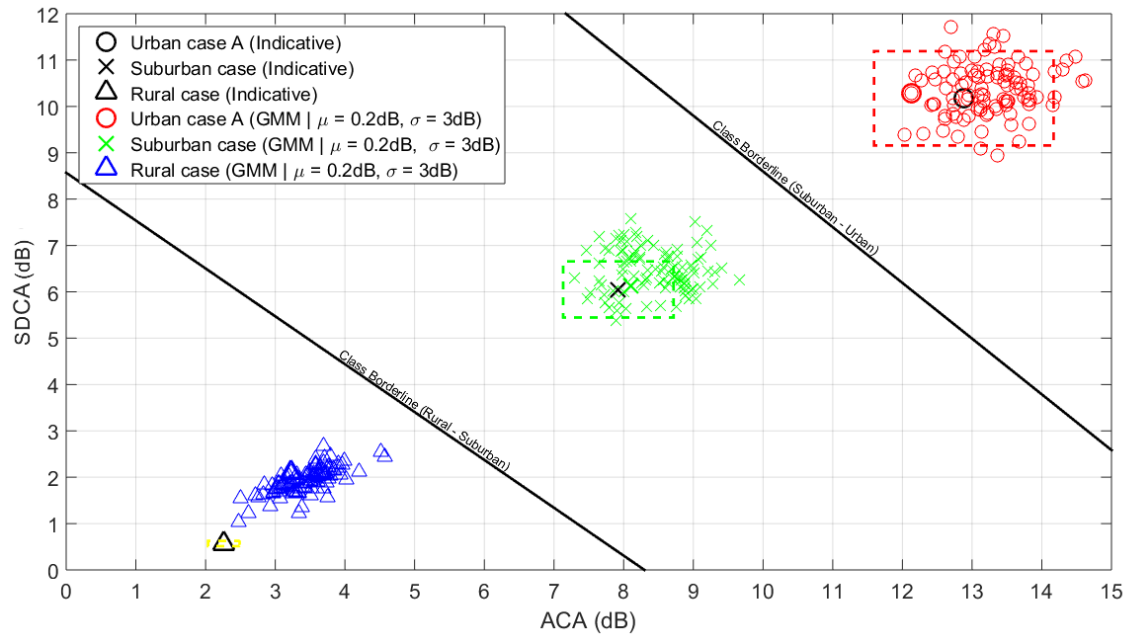
Combining findings of Figures 3(a)-3(e) with Table 3, it is shown that similarly to the ND mean case, where the class consistency did not remain intact in a concentrated way, the variation of ND standard deviation σ demonstrates a more disruptive influence on the GMM virtual OV LV BPL topologies. While at $\sigma = 0$ dB and 1dB all topologies preserve full class membership (100% inside-the-class), box compliance already begins to decline, most notably for rural at $\sigma = 1$ dB (1%). As σ increases to 3dB, class integrity is still maintained (100%), yet compatibility with the $\pm 10\%$ ACA / SDCA box drops sharply (0%, 37%, 85% for rural, suburban and urban A respectively), signaling the onset of spatial dispersion. Beyond this point, for $\sigma \geq 5$ dB, the rural and suburban cases no longer remain fully inside their classes (87% and 99% at $\sigma = 5$ dB, falling further to 31% and 79% at $\sigma = 7$ dB), indicating cross-class GMM virtual OV LV BPL topology footprint migration, whereas the urban A topology retains 100% class consistency throughout due to the reason discussed in the previous subsection but exhibits significant reduction in the GPI inside-the-box compliance (65% at $\sigma = 5$ dB and 30% at $\sigma = 7$ dB). Overall, increasing σ expands the footprint spatially, progressively reducing GPI inside-the-box percentages across all cases, with rural being the most sensitive and urban A the most resilient to dispersion effects thus validating the observations of Figures 3(a)-3(e).



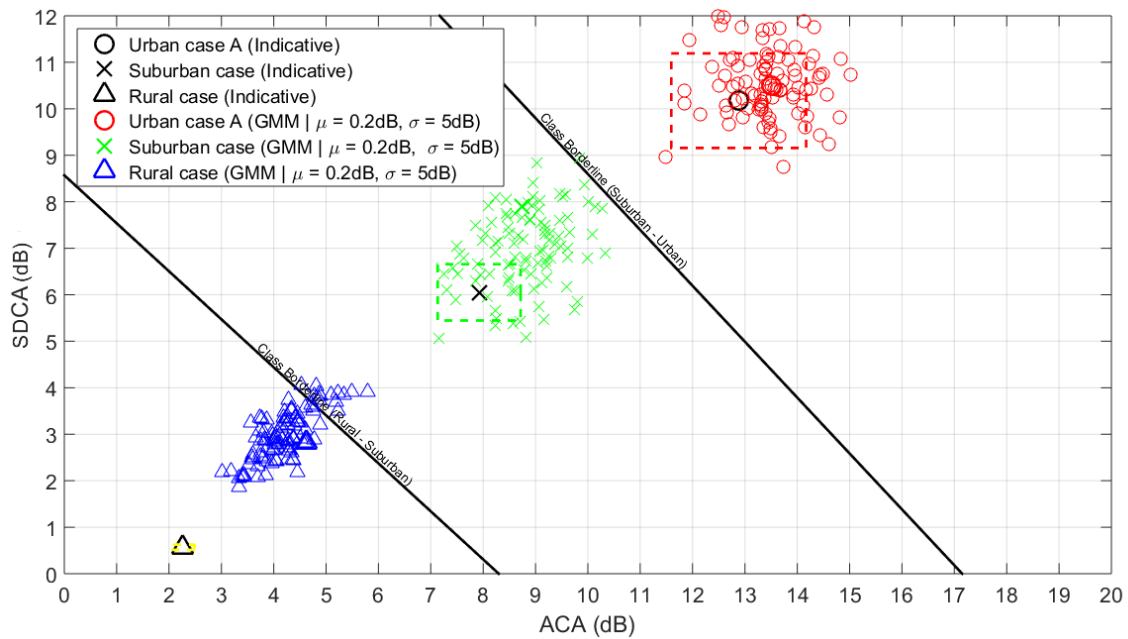
(a)



(b)



(c)



(d)

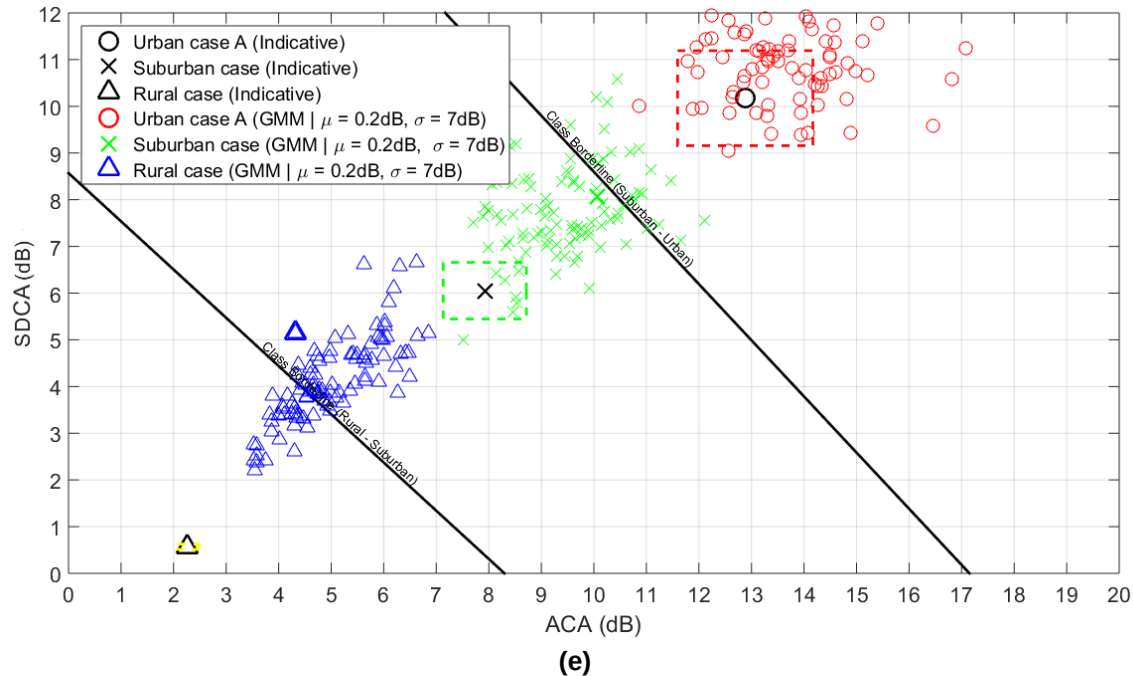


Fig. 3. OV LV BPL topology class map with GMM virtual topology footprints of the three indicative OV LV BPL topologies for various single ND standard deviations. **(a)** ND standard deviation $\sigma = 0\text{dB}$. **(b)** ND standard deviation $\sigma = 1\text{dB}$. **(c)** ND standard deviation $\sigma = 3\text{dB}$. **(d)** ND standard deviation $\sigma = 5\text{dB}$. **(e)** ND standard deviation $\sigma = 7\text{dB}$.

Table 3. GPIs of the GMM OV LV BPL topologies of the three OV LV BPL topology classes for different ND standard deviation σ values when the ND mean μ is equal to 0.2dB . (GMM default operation settings are applied).

ND standard deviation $\sigma = 0\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	100	100	100
ND standard deviation $\sigma = 1\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	1	100	100
ND standard deviation $\sigma = 3\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	0	37	85

ND standard deviation $\sigma = 5\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	87	99	100
Inside the box (%)	0	16	65

ND standard deviation $\sigma = 7\text{dB}$			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	31	79	100
Inside the box (%)	0	6	30

As the impact of the ND standard deviation σ is examined on the population of OV LV BPL topology classes, the population mechanism transitions from the primarily horizontal ACA-oriented spreading observed under ND mean variation to a broader, diagonal dispersion pattern across the ACA / SDCA plane. Increasing σ not only shifts the GMM virtual OV LV BPL topologies toward higher ACA and SDCA values simultaneously, but also expands their spatial footprint, enabling population of topological regions of OV LV BPL topology class map that were previously inaccessible when μ dominated the population process. Unlike the targeted, axis-aligned population behavior driven by μ , where GMM virtual OV LV BPL topology footprints migrate mainly parallel to ACA while maintaining similar SDCA elevation, the σ -driven mechanism populates regions diagonally, thus covering alternative regions of the OV LV BPL topology class map with enhanced spatial reach. This complementary behavior implies that, through appropriate selection and mixing of NDs within the GMM, population can be strategically directed either horizontally (via μ) or diagonally with increased area coverage (via σ), enabling controlled sampling of distinct topological zones across rural, suburban and urban classes.

Comparing Figure 1(b) and Table 1 of NNTGM with Figures 3(a)-3(e) and Table 3 of GMM, respectively, the different population approaches of NNTGM and GMM are again verified. Although both methods consistently achieve over 80% GPI inside-the-box for the rural, suburban and urban cases, confirming the stable class preservation when the ND mean and ND standard deviations of GMM remain low, the more abrupt, almost step-like response for GMM also occurs even if ND standard deviation little increases (e.g., for ND standard deviation values greater than 1dB). Anyway, the same 100% GPI inside-the-class occurs for NNTGM and GMM regardless of the ND standard deviation value for GMM when the ND mean remains low.

Total Count of NDs and its Impact on GMM Virtual Topology Footprints and GPIs

Up to this point, the GMM has been examined consisting of only one ND. In this context, the influence of the ND mean μ and the ND standard deviation σ was investigated. In this subsection, the effect of the number of NDs that can be incorporated into a GMM is examined. Using the previously obtained ND results, GMMs are constructed in the form of scenarios, as presented below:

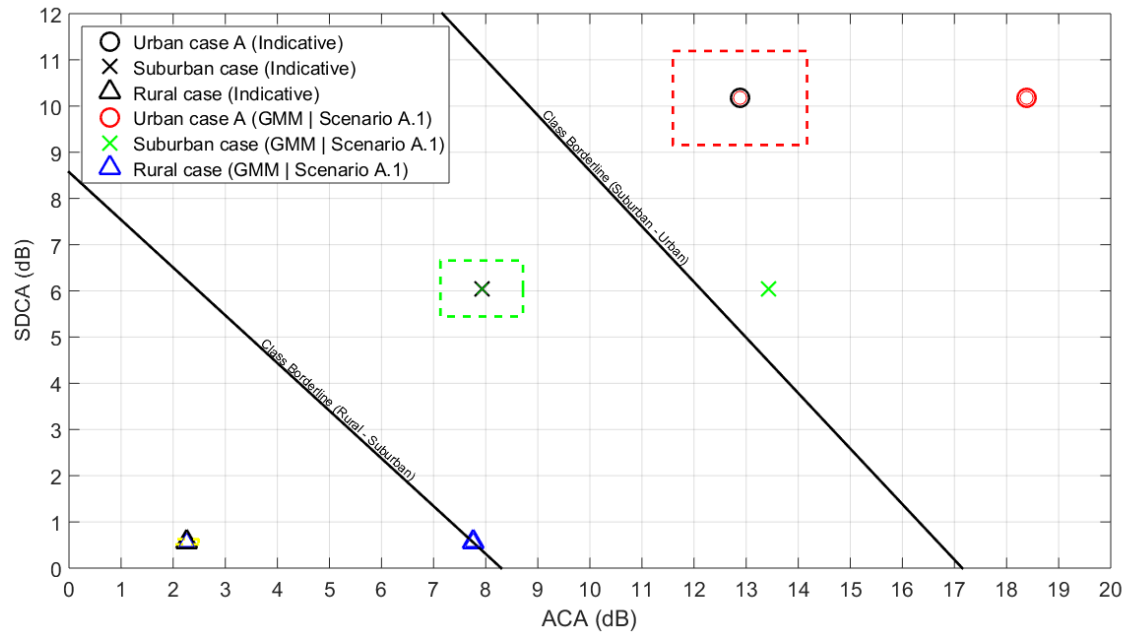
- *Scenario A.1:* GMM of Scenario A.1 consists of 2 ND components. The first ND of this scenario is characterized by its mean $\mu_{A.1.1}$ and standard deviation $\sigma_{A.1.1}$

- that are equal to 0dB and 0.5dB, respectively, -see Figure 2(a)- with weight $w_{A.1.1} = 0.5$. The second ND of this scenario is characterized by its mean $\mu_{A.1.2}$ and standard deviation $\sigma_{A.1.2}$ that are equal to 5.5dB and 0.5dB, respectively, -see Figure 2(e)- with weight $w_{A.1.2} = 0.5$.
- *Scenario A.2:* GMM of Scenario A.2 consists of 3 ND components. The first ND of this scenario is characterized by its mean $\mu_{A.2.1}$ and standard deviation $\sigma_{A.2.1}$ that are equal to 0.2dB and 0.5dB, respectively, with weight $w_{A.2.1} = 0.34$. The second ND of this scenario is characterized by its mean $\mu_{A.2.2}$ and standard deviation $\sigma_{A.2.2}$ that are equal to 0.2dB and 3dB, respectively, -see Figure 3(c)- with weight $w_{A.2.2} = 0.33$. The third ND of this scenario is characterized by its mean $\mu_{A.2.3}$ and standard deviation $\sigma_{A.2.3}$ that are equal to 0.2dB and 7dB, respectively, -see Figure 3(e)- with weight $w_{A.2.3} = 0.33$.
 - *Scenario A.3:* GMM of Scenario A.3 consists of 2 ND components. The first ND of this scenario is characterized by its mean $\mu_{A.3.1}$ and standard deviation $\sigma_{A.3.1}$ that are equal to 3dB and 0.5dB, respectively, -see Figure 2(d)- with weight $w_{A.3.1} = 0.5$. The second ND of this scenario is characterized by its mean $\mu_{A.3.2}$ and standard deviation $\sigma_{A.3.2}$ that are equal to 0.2dB and 5dB, respectively, -see Figure 3(d)- with weight $w_{A.3.2} = 0.5$.

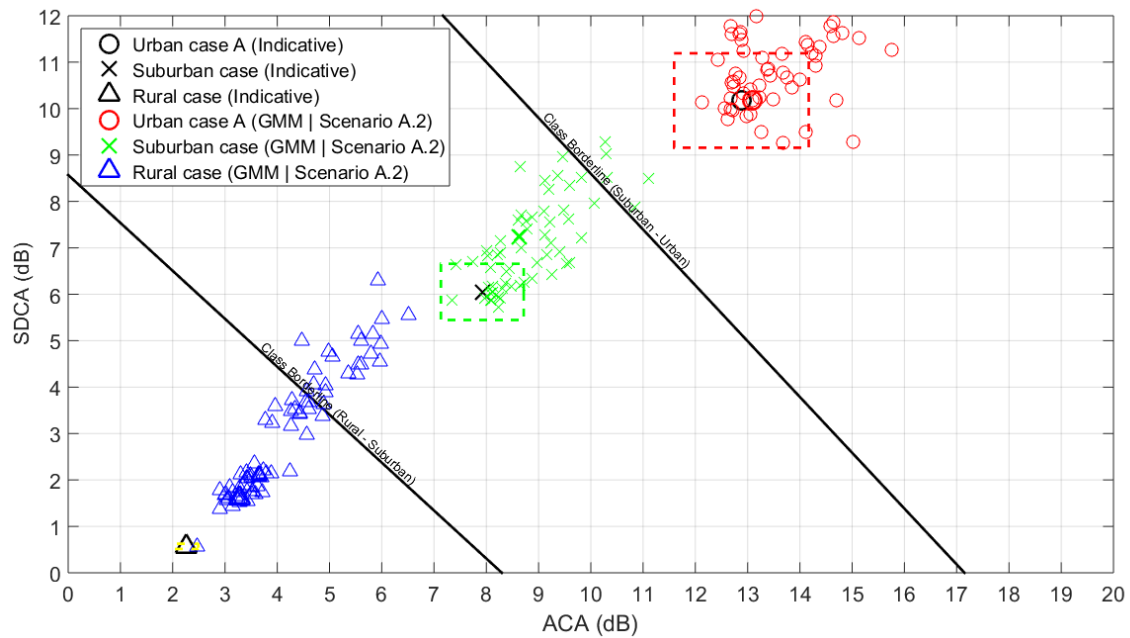
As the impact of the total count of NDs is concerned in the operation of GMM, in Figure 4(a), the OV LV BPL topology class map with the GMM virtual topology footprints of the rural case, suburban case and urban case A is presented when the GMM default operation settings are applied and GMM consists of the NDs of the Scenario A.1. Each GMM virtual OV LV BPL topology footprint of Figure 4(a) comprises 100 GMM virtual topologies per indicative OV LV BPL topology. In Figs 4(b) and 4(c), similar plots to Figure 4(a) are given, but for the case of Scenario A.2 and Scenario A.3, respectively. The same GPIs of Tables 2 and 3 are applied to the GMM virtual OV LV BPL topologies of the three OV LV BPL topology classes of Figures 4(a)-4(c) and are reported in Table 4.

Observing Figures 4(a)-4(c), the impact of the number of NDs participating in the GMM construction is investigated under various scenarios of ND components, namely:

- Under Scenario A.1, two narrow ND components with equal weights are superposed. The resulting GMM generates, for each indicative OV LV BPL topology, a pair of compact virtual OV LV BPL topology footprint clusters aligned horizontally along the ACA axis and separated by approximately 5.5dB. Because both ND components are characterized by low values of standard deviation, the within-cluster spread is minimal and the SDCA coordinate remains essentially unchanged, so the footprints appear as two tight modes: one centered close to the indicative topology and a second one translated to higher ACA by approximately 5.5dB. This horizontal bimodality is consistently observed across the rural, suburban and urban A cases and preserves the class separability structure seen in the underlying NDs. Notably, the right shifted modes bring the suburban footprint locally across the suburban-urban class borderline, while the rural footprint exceeds the rural-suburban class borderline boundary highlighting that small systematic ACA biases can trigger class relabeling even when SDCA remains stable. Table 4 reports GPIs that align with the findings from Figure 4(a);



(a)



(b)

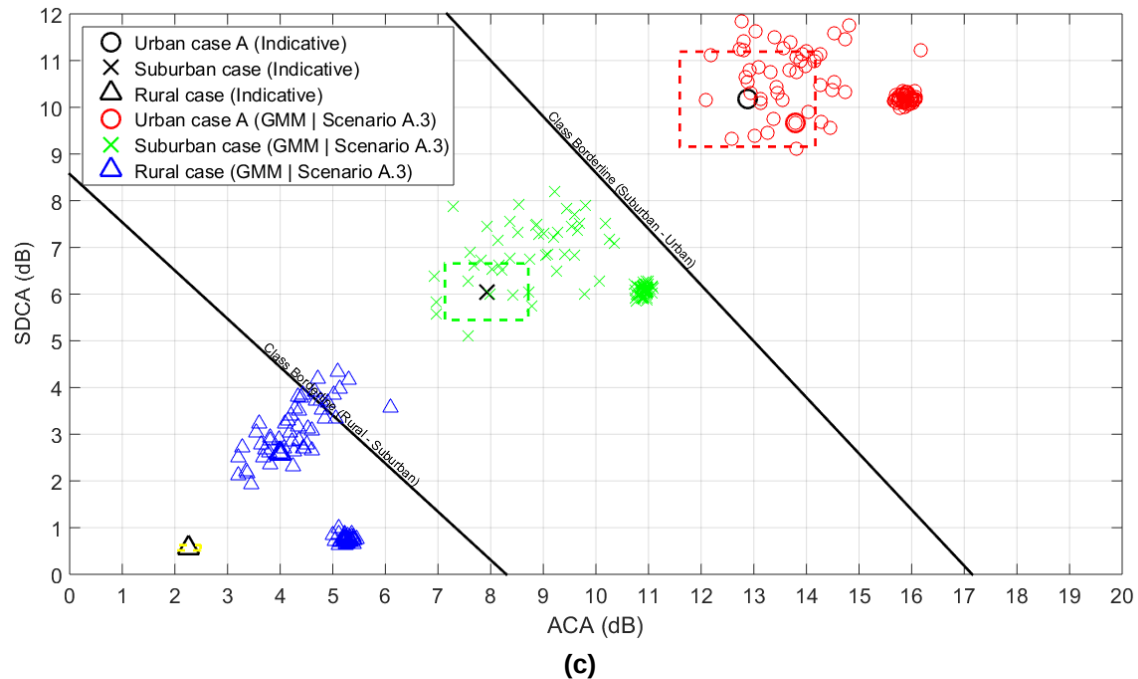


Fig. 4. OV LV BPL topology class map with GMM virtual topology footprints of the three indicative OV LV BPL topologies for various total counts of NDs. **(a)** Scenario A.1 (2 NDs). **(b)** Scenario A.2 (3 NDs). **(c)** Scenario A.3 (2 NDs).

Table 4. GPIs of the GMM OV LV BPL topologies of the three OV LV BPL topology classes for different scenarios of total count of NDs (GMM default operation settings are applied).

Scenario A.1 (2 NDs)			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	57	55	100
Inside the box (%)	57	55	49

Scenario A.2 (3 NDs)			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	78	95	100
Inside the box (%)	25	57	69

Scenario A.3 (2 NDs)			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	91	100	100
Inside the box (%)	0	7	26

for the rural and suburban cases, the right-shifted mode grazes the class boundary, yielding GPI inside-the-class values of 57% and 55%, respectively, while the corresponding GPI inside-the-box values are also 57% and 55%. This outcome is expected, given that weights $w_{A.1.1} = 0.5$ and $w_{A.1.2} = 0.5$ were adopted. In particular, the first weight $w_{A.1.1}$, which favors tightly concentrated modes around the indicative OV LV BPL topologies under examination, results through the simulation process in approximately 50 out of the 100 generated GMM virtual OV LV BPL topologies falling within the GPI inside-the-box region for the all the indicative OV LV BPL topologies. For urban, both modes remain within class (100% GPI inside-the-class), but the shifted mode sits outside the local box, reducing the GPI inside-the-box to 49%. In this sense, multi-ND GMMs increase representation granularity while retaining interpretable class boundaries.

- In Scenario A.2, three ND components with (approximately) the same mean μ of 0.2dB and varying ND standard deviation values σ of 0.5dB, 3dB and 7dB are combined. Because the components are heteroscedastic but not mean-shifted, each indicative OV LV BPL topology becomes the anchor of a multi-scale footprint: a tight core ($\sigma_{A.2.1}=0.5\text{dB}$), a medium-spread halo ($\sigma_{A.2.2}=3\text{dB}$) and an extended tail ($\sigma_{A.2.3}=7\text{dB}$). In the ACA / SDCA plane, the resulting virtual OV LV BPL topologies organize along a diagonal manifold so that the three variance levels appear as an idiosyncratic diagonal ‘stitching’ of sub-footprints rather than a purely horizontal broadening. Compared with Scenario A.1 (which primarily controls position through bimodal ACA shifts), Scenario A.2 primarily controls the spatial extent (dispersion) of the footprint. Practically, this allows population to be allocated to targeted regions of the OV LV BPL topology class map and, by tuning the number / variances of the contributing NDs, to regulate the parameters of the footprint. As visible in Figure 4(b), the virtual OV LV BPL topology footprint for suburban case elongates toward the suburban-urban class borderline and the rural footprint toward the rural-suburban one: the bulk remains in-class, yet the high-variance tail increases the likelihood of boundary grazing or crossings, for rural and suburban cases. Quantitatively, the GPI indicators in Table 4 confirm the inside-the-class and inside-the-box behavior for the rural, suburban and urban cases.
- To regulate simultaneously both the degree of concentration at the core of the footprint and the rate at which the footprint becomes more diffuse toward its outer regions, Scenario A.3 combines two ND components belonging to different footprint families (in contrast to Scenarios A.1 and A.2). This configuration makes it possible to form either a tightly concentrated mode surrounded by a more loosely distributed outer region or, alternatively, a locally dense area from which a gradually sparse distribution emerges. Specifically, it uses: (i) a high-mean, low-variance component ($\mu_{A.3.1}=+3\text{dB}$, $\sigma_{A.3.1}=0.5\text{dB}$, see Figure 2(d)) with weight $w_{A.3.1} = 0.5$, and (ii) a near-unbiased, high-variance component ($\mu_{A.3.2}=0.2\text{dB}$, $\sigma_{A.3.2}=5\text{dB}$, see Figure 3(d)) with weight $w_{A.3.2}=0.5$. The resulting GMM produces distinctly bimodal virtual OV LV BPL topology footprints: a compact, right-shifted nucleus (approximately +3dB along ACA) and a diffuse cloud centered relatively near the indicative topology. Geometrically, the compact mode captures the horizontal displacement trend characteristic of Scenario A.1, while the diffuse mode unfolds along the diagonal ACA–SDCA manifold

associated with variance-driven spread in Scenario A.2. In practice, this configuration allows co-control of position and dispersion: the tightly concentrated mode can be subtly shifted to control the location of the most densely populated region of the footprint, whereas the loosely concentrated, high-variance mode governs the overall spatial spread of the footprint by distributing probability mass. As seen in Figure 4(c), the virtual OV LV BPL topology footprint for suburban case exhibits a small dense core close to the suburban-urban class borderline together with a broader cloud around the indicative point, increasing the chance of boundary grazing under mild ACA biases; the rural footprint retains most mass in-class while permitting occasional excursions toward the rural-suburban class borderline; the urban footprint remains within its class region. Overall, Scenario A.3 inherits favorable properties from both Scenarios A.1 and A.2, enabling population allocation across multiple regions of the class map with differentiated extent. This translates into controlled footprint compactness and tunable class interaction, a behavior that is consistent with the comparative trends reported in Tables 2–4, where the resulting GPI values are driven by the chosen mixture and the spatial placement of the two constituent ND components.

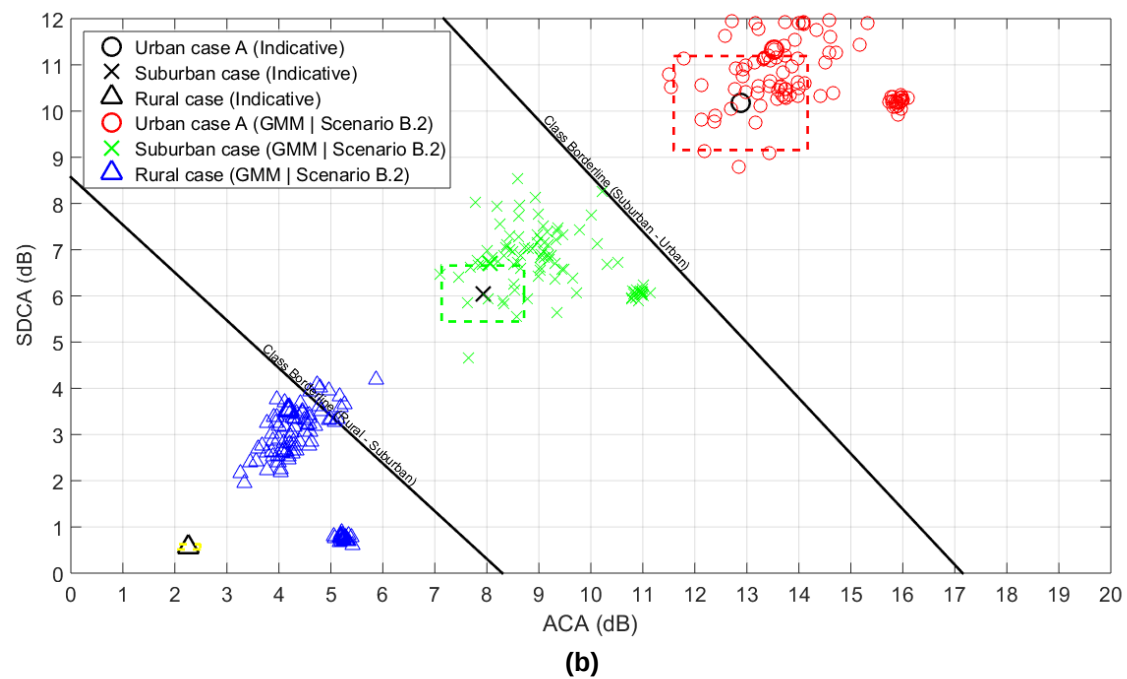
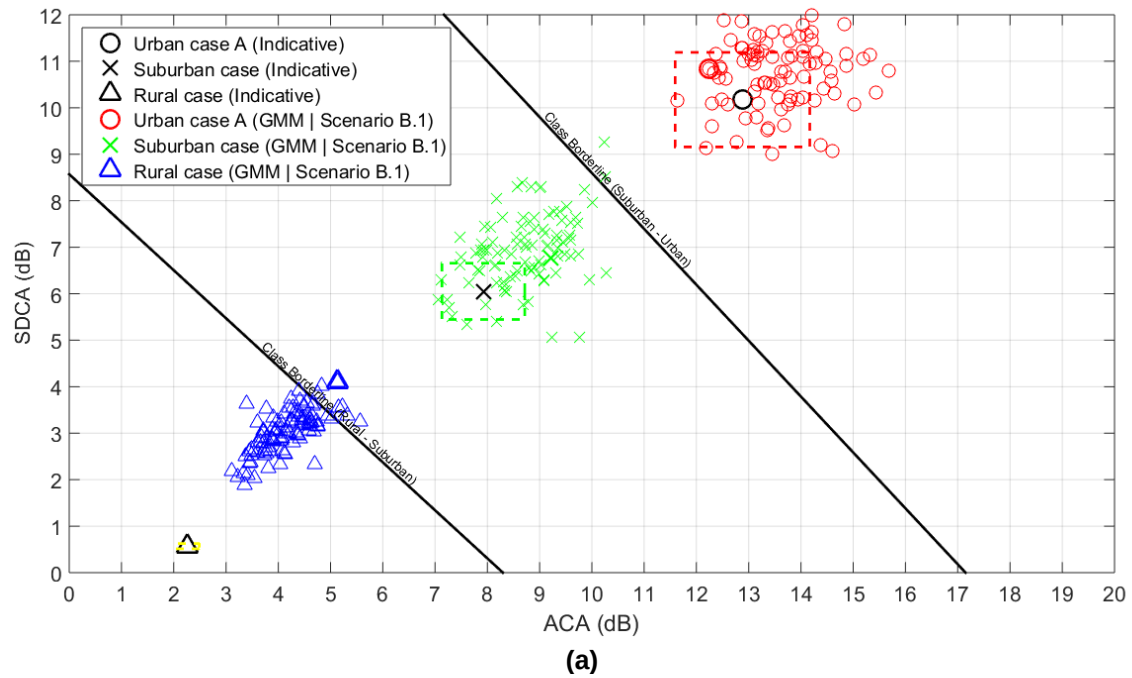
In this subsection, the ND components included in the Scenarios A.1-A.3 contributed with equal weighting. It is evident that increasing the number of NDs within the GMM modifies both the locally dense regions and the more diffuse outer regions of the resulting GMM virtual OV LV BPL topology footprints. By enriching the mixture with additional tightly and loosely concentrated modes, the GMM gains the flexibility to populate previously unoccupied areas of the ACA / SDCA space. Among the Scenarios A.1-A.3 examined herein, Scenario A.3 demonstrated the greatest versatility, as it combines NDs of different footprint behavior families with equal weights. For this reason, the versatility of Scenario A.3 is further examined in the following subsection, where the potential for footprint displacements is investigated under non-uniform weight allocation rather than equal weighting. Moreover, by departing from uniform weights, the analysis also explores the extent to which the density distribution of the more diffuse outer regions of the resulting GMM virtual OV LV BPL topology footprints can be modulated, thus revealing the additional degrees of control offered by asymmetric weighting.

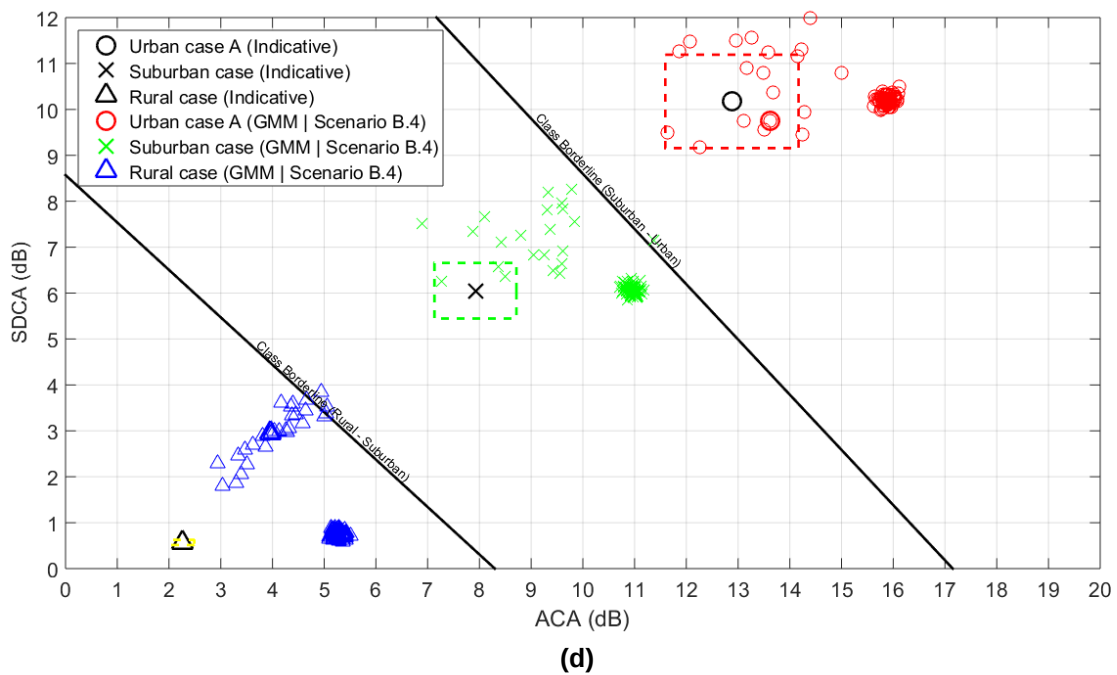
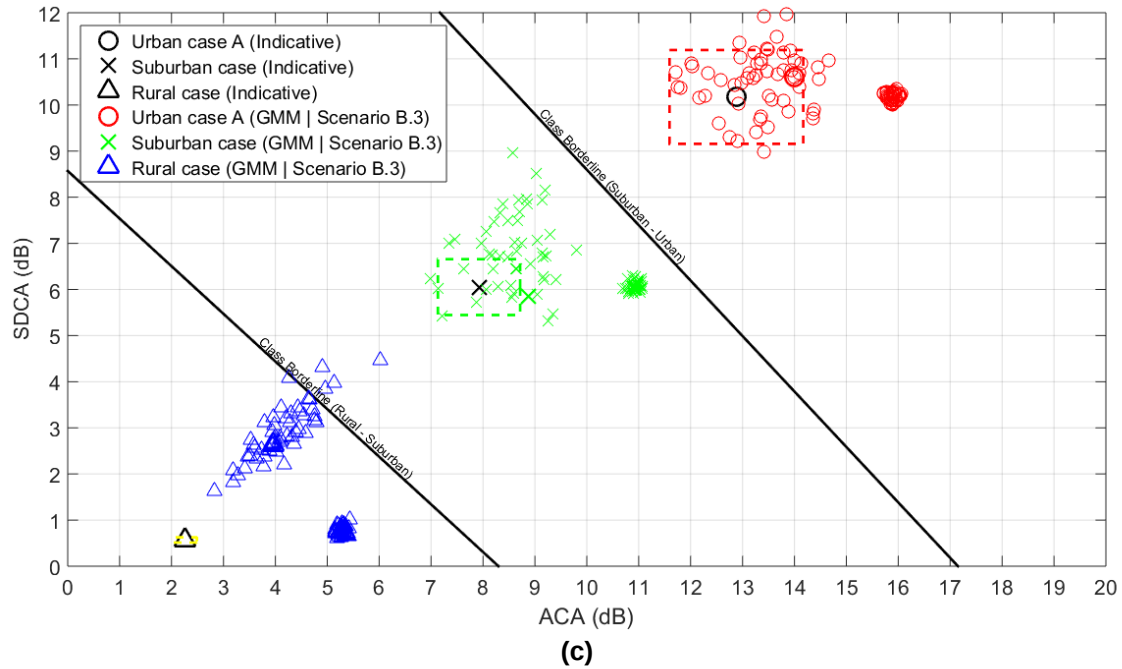
ND Weighting and their Impact on GMM Virtual Topology Footprints and GPIs

Building upon the findings of Scenarios A.1-A.3 of the previous subsection, this subsection focuses on Scenario B, in which the ND components no longer participate with equal weighting. Instead, selected non-uniform weight distributions are applied to the NDs previously used in Scenario A.3 in order to investigate how asymmetric weighting influences both the positioning and the dispersion behavior of the resulting GMM virtual OV LV BPL topology footprints. By varying the contribution of each ND, the analysis evaluates not only the degree of controllability in adjusting the location of the most densely concentrated region but also the ability to modulate the density of the more diffuse outer regions of the footprint. In this way, the study extends the versatility demonstrated in Scenario A.3 by revealing how shifts in weight allocation can simultaneously affect footprint displacement and the structure of its concentrated and diffuse modes. Similarly to the previous subsection, GMMs are constructed in the form of scenarios, as presented below:

- *Scenario B.1:* This GMM consists of 2 ND components. The first ND of this scenario is characterized by its mean $\mu_{B.1.1}$ and standard deviation $\sigma_{B.1.1}$ that are equal to 3dB and 0.5dB, respectively, with weight $w_{B.1.1} = 0$. The second ND of this scenario is characterized by its mean $\mu_{B.1.2}$ and standard deviation $\sigma_{B.1.2}$ that are equal to 0.2dB and 5dB, respectively, with weight $w_{B.1.2} = 1$. Practically, the Scenario B.1 consists of the same NDs (i.e., identical ND mean and ND standard deviation values) as those used in Scenario A.3, but with different weight assignments. Specifically, selecting $w_{B.1.2} = 1$ implies that only the second ND is considered, leading the GMM virtual OV LV BPL topology footprint to degenerate into the analysis of a similar footprint to one shown in Figure 3(d). Although the second ND in this scenario is going to be identical to that employed in Figure 3(d) in terms of ND mean and ND standard deviation, minor deviations are expected due to the inherent randomness of the sampling process. Nevertheless, these deviations are expected to remain limited and further validate the robustness of the proposed methodology.
- *Scenario B.2:* This scenario is based on Scenario B.1, with the only difference being that it applies weights $w_{B.2.1} = 0.25$ and $w_{B.2.2} = 0.75$ for the same NDs of Scenario B.1. This weight configuration is introduced in order to examine the effect of varying the ND weights within the GMM, while keeping the ND components fixed.
- *Scenario B.3:* Following the rationale of Scenarios B.1 and B.2, Scenario B.3 employs weights $w_{B.3.1} = 0.50$ and $w_{B.3.2} = 0.50$ for the same NDs of Scenarios B.1 and B.2 in order to investigate the effect of varying the ND weights within the GMM, consistent with the analysis conducted in Scenario B.2. In fact, this particular weight combination corresponds to the case of Scenario A.3 illustrated in Figure 4(c). However, as in Scenario B.1 in comparison with Figure 4(c), minor deviations may arise due to the inherent randomness of the sampling process. Nonetheless, these deviations are expected to remain limited and further substantiate the robustness of the proposed methodology.
- *Scenario B.4:* Same with Scenario B.1 but with applied weights $w_{B.4.1} = 0.75$ and $w_{B.4.2} = 0.25$ for the same NDs of Scenario B.1.
- *Scenario B.5:* This scenario is structurally identical to Scenario B.1; however, it represents its counterpart in terms of weight allocation. Specifically, in Scenario B.5, the weights are assigned as $w_{B.5.1} = 1$ and $w_{B.5.2} = 0$. This configuration implies that only the first ND is effectively considered in the analysis. Consequently, the resulting GMM virtual OV LV BPL topology footprint degenerates into a case analogous to that previously examined in Figure 2(d). Although the first ND in this scenario is intended to replicate the characteristics of the ND employed in Figure 2(d) (i.e., identical mean and standard deviation values), minor deviations are anticipated due to the inherent randomness of the sampling process. Nevertheless, these deviations are expected to remain limited and further substantiate the robustness of the proposed methodology.

As the impact of the ND weights is concerned in the operation of GMM, in Figure 5(a), the OV LV BPL topology class map with the GMM virtual topology footprints of the rural case, suburban case and urban case A is presented when the GMM default operation settings are applied and GMM consists of the NDs of the Scenario B.1.





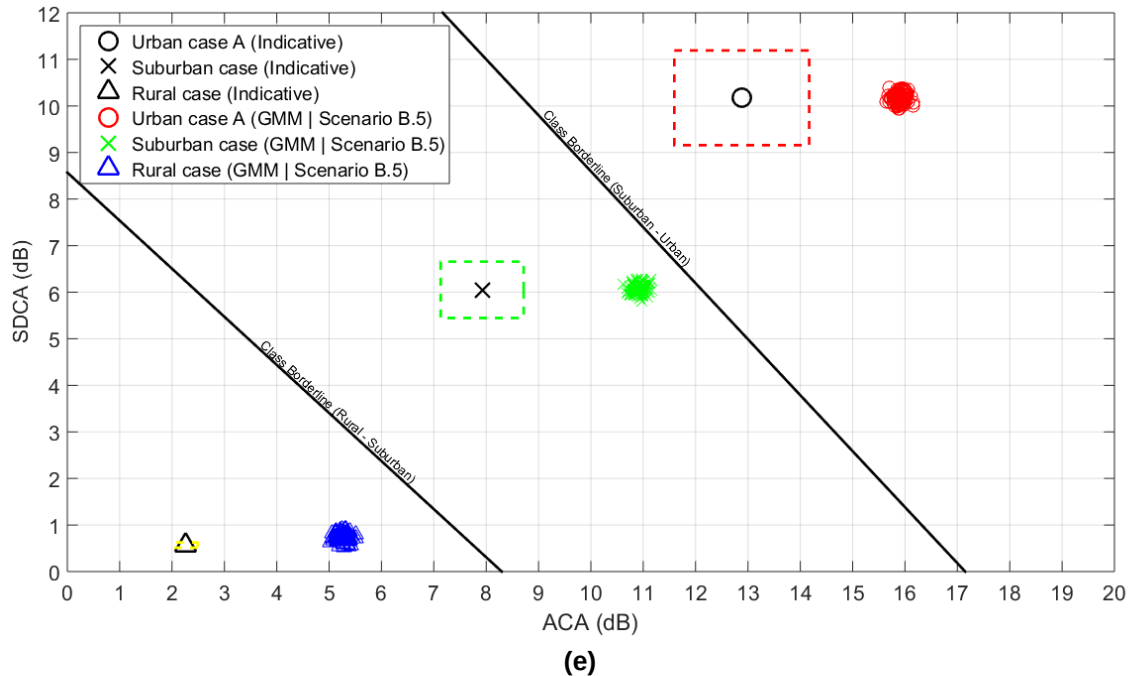


Fig. 5. OV LV BPL topology class map with GMM virtual topology footprints of the three indicative OV LV BPL topologies for various ND weights. **(a)** Scenario B.1: $-w_{B.1.1} = 0$ and $w_{B.1.2} = 1$, like Figure 3(d)-. **(b)** Scenario B.2 ($w_{B.2.1} = 0.25$ and $w_{B.2.2} = 0.75$). **(c)** Scenario B.3 - $w_{B.3.1} = 0.5$ and $w_{B.3.2} = 0.5$, like Figure 4(c)-. **(d)** Scenario B.4 ($w_{B.4.1} = 0.75$ and $w_{B.4.2} = 0.25$). **(e)** Scenario B.5 - $w_{B.5.1} = 1$ and $w_{B.5.2} = 0$, like Figure 2(d)-.

Each GMM virtual OV LV BPL topology footprint of Figure 5(a) comprises 100 GMM virtual topologies per indicative OV LV BPL topology. In Figures 5(b)-5(e), similar plots to Figure 5(a) are given, but for the case of Scenarios B.2-B.5, respectively. The same GPIs of Tables 2 and 3 are applied to the GMM virtual OV LV BPL topologies of the three OV LV BPL topology classes of Figures 5(a)-5(e) and are reported in Table 5.

Figures 5(a)-5(e) illustrate how ND weighting reshapes the GMM virtual OV LV BPL topology footprints when interpolating between two distinct footprint families: a near-unbiased, high-variance ND that generates a loosely concentrated, diagonally oriented diffuse distribution similar to Figure 3(d), and a low-variance, +3dB ACA ND that produces a tightly concentrated, horizontally aligned cluster comparable to Figure 2(d). In Scenario B.1 ($w_{B.1.1} = 0$, $w_{B.1.2} = 1$), the footprints manifest primarily as diffuse diagonal clouds anchored close to the indicative OV LV BPL topology in each class. As the weight of the compact +3dB component increases in Scenario B.2 ($w_{B.2.1} = 0.25$, $w_{B.2.2} = 0.75$) and Scenario B.3 ($w_{B.3.1} = 0.5$, $w_{B.3.2} = 0.5$) to Scenario B.4 ($w_{B.4.1} = 0.75$, $w_{B.4.2} = 0.25$), a tightly concentrated right-shifted cluster begins to emerge and gradually dominates the footprint. Concurrently, the loosely concentrated diffuse region loses probability mass and becomes progressively less pronounced. This coordinated reallocation of density causes the footprint's most populated region to shift toward higher ACA values while the overall geometric extent contracts. At the extreme of Scenario B.5 ($w_{B.5.1} = 1$, $w_{B.5.2} = 0$), each class collapses into a compact, right-shifted cluster with minimal change in SDCA, positioned well within the boundaries of its respective class region.

Table 5. GPIs of the GMM OV LV BPL topologies of the three OV LV BPL topology classes for different scenarios of ND weights (GMM default operation settings are applied).

Scenario B.1			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	93	98	100
Inside the box (%)	0	20	48

Scenario B.2			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	91	100	100
Inside the box (%)	0	10	41

Scenario B.3			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	96	100	100
Inside the box (%)	0	11	43

Scenario B.4			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	97	99	100
Inside the box (%)	0	0	0

Scenario B.5			
GPI	OV LV BPL topology class		
	Rural case class	Suburban case class	Urban case A class
Inside the class (%)	100	100	100
Inside the box (%)	0	0	0

The rebalancing between a diagonal, high-variance near-unbiased mode and a compact, right-shifted mode of Figures 5(a)-5(e) may also be reflected in the GPIs of Table 5. The GPI inside-the-class is already high in Scenario B.1 (93% for the rural case, 99% for the suburban case and 100% for the urban case A). Actually, the GPI inside-the-class never falls below 91% across all weightings. As the compact component becomes more influential in Scenarios B.2 through B.5, boundary grazing subsidies and the GPI inside-the-class GPI converges to 100% for all three classes in Scenario B.5, with the urban case remaining at 100% throughout and the suburban case reaching 100% from Scenario B.2 onward. By contrast, the GPI inside-the-box shows the complementary behavior because the compact component is steadily displaced from the indicative OV LV BPL topology in each case. As its weight increases, probability mass migrates away

from the local boxes: for the suburban case the values decline from 20% in Scenario B.1 to around 10% to 11% percent in Scenarios B.2 and B.3, and then to 0% in Scenarios B.4 and B.5; for the urban case the values drop from 48% in Scenario B.1 to roughly 41% to 43% in Scenarios B.2 and B.3, and then to 0% in Scenarios B.4 and B.5; for the rural case the value is already 0% in Scenario B.1 and remains at 0% thereafter, indicating that even the diffuse configuration falls outside the tight indicative box. Overall, Figures 5(a)-5(e) together with the GPIs of Table 5 demonstrate that the transformations observed under varying weight allocations correspond to a redistribution of probability mass between two coexisting modes rather than a rotation or continuous geometric deformation of a single shape. What changes is not the fundamental structure of the modes but the balance of density between the tightly concentrated cluster and the loosely concentrated diffuse one. As the weights vary, the diffuse outer region may retain significant mass, become partially depleted, or nearly vanish showing that its density is directly controllable through the weighting scheme.

This interpretation naturally extends to mixtures incorporating more than two ND components: both the resulting footprints and their corresponding GPIs behave as weighted superpositions whose attainable spatial patterns are bounded by the geometry and the GPI ranges of the constituent modes. Consequently, the possible outcomes remain confined within the limits set by these components, while non-uniform weights allow fine-grained regulation of how probability mass is allocated between dense and diffuse regions, further enriching the modeling flexibility of the framework.

Discussion and Future Research

This paper has established a structured examination of virtual OV LV BPL topology footprints within the GMM framework. Since the conceptual basis for applying GMM to populate the corresponding class maps through the fundamentals of class maps, topology footprints and GPIs for OV LV BPL topologies, an analysis of how specific GMM parameters —namely the mean and standard deviation of the NDs, the total number of NDs and the weight assigned to each ND— affect the resulting GMM virtual OV LV BPL topology footprints and GPIs has been given.

Building on these contributions, three avenues for future research emerge, namely:

- Further investigation is required to establish a rigorous methodology for correlating both the centroid and the spatial extent of the near Minimum Volume Enclosing Ellipsoid (MVEE) approximation of the GMM virtual OV LV BPL topology footprints and clusters with the underlying MVEE parameters. This includes exploring how variations in (i) the ellipsoid center; (ii) the lengths of the semi-major and semi-minor axes; and (iii) the overall volume of the ellipsoid collectively shape the resulting footprint geometry. This includes exploring how variations in (i) the mean μ of the NDs, (ii) the standard deviation σ of the NDs, (iii) the total count of NDs incorporated in GMM, and (iv) the assigned ND weights; collectively shape the resulting footprint geometry. Developing such a correlation framework would substantially deepen our understanding of parameter sensitivity and structural behavior within MVEE-based representations and GMM.
- Future efforts should aim to identify and refine techniques for detecting the center and extent of virtual OV LV BPL topology footprints and clusters, whether obtained through GMM or through alternative modeling methodologies such as the NNTGM.

- An important direction for forthcoming research is the development of systematic procedures for replicating virtual OV LV BPL topology footprints using GMM, whether obtained through GMM or through alternative modeling methodologies such as the NNTGM. Establishing reliable replication mechanisms will be essential for validating model behavior, supporting scenario-based analyses and enabling broader applicability of virtual topology reconstruction in future BPL network studies.

Conclusions

In this paper, the theory and relevant simulation results for populating OV LV BPL topology classes by applying GMM have been given. With reference to the class maps and the GPIs, the impact of the following GMM parameters on the GMM virtual OV LV BPL topology footprints has been assessed, namely: (i) the mean μ of the ND components of GMM, (ii) the standard deviation σ of the NDs, (iii) the total count of NDs incorporated in the mixture, and (iv) the assigned ND weights. First, as the impact of the ND mean μ is examined, GMM virtual footprints can populate topological zones as μ increases, maintaining similar SDCA elevations and remaining approximately parallel along the ACA axis. Second, as the impact of the ND standard deviation σ is examined, the population mechanism of GMM virtual OV LV BPL topology footprints shifts from the primarily horizontal, μ -driven spreading of OV LV BPL topologies to a broader, diagonal dispersion across the ACA / SDCA plane of the OV LV BPL topology class maps, simultaneously increasing both dimensions. At the same time, increasing σ expands the GMM virtual topology footprint and enables population of previously inaccessible regions, thus providing a complementary mechanism to the more axis-aligned population behavior induced by the ND mean μ . Third, regarding the impact of the number of NDs in the GMM, increasing the number of ND components does not simply shift centroid positions or broaden the spatial coverage of the resulting GMM virtual OV LV BPL topology footprints. Instead, the addition of extra NDs enhances the representational richness of the mixture by introducing new footprint behavior families ranging from tightly concentrated to loosely concentrated modes. This expanded set of constituent behaviors allows the GMM to generate more complex superpositions and to form composite footprints whose structure results from the interaction of multiple concentrated and diffuse sub-footprints. Consequently, the attainable patterns in the ACA / SDCA domain are defined not by arbitrary expansion but by the geometric and probabilistic envelopes established by the added modes. Fourth, regarding the impact of ND weights in the GMM, beyond merely including more ND components, adjusting their weights provides precise control over how probability mass is redistributed between the dense core and the more diffuse outer regions of the footprint. Rather than affecting only centroid trajectories or simple dispersion metrics, non-uniform weight allocation governs the prominence, attenuation or suppression of each sub-footprint. This enables deliberate shaping of both the intensity of the tightly concentrated region and the density profile of the loosely concentrated diffuse region, allowing smooth transitions between footprint forms and offering finer control than equal-weight mixtures. Through weighting, the GMM can therefore regulate footprint structure rather than merely repositioning it. Concluding this paper, three avenues for future research have been recognized so that the practicality of the GMM should be enhanced to function either as a detector of virtual OV

LV BPL topology footprints whether derived from GMM or alternative modeling methodologies such as NNTGM or as a replicator of such footprints, regardless of their origin from GMM or other approaches like NNTGM.

CONFLICTS OF INTEREST

The author declares that there is no conflict of interests regarding the publication of this paper.

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